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Dynamical stability and evolution of the discs of Sc galaxies

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ABSTRACT

We examine the local stability of galactic discs against axisymmetric density perturbations with special attention to the different dynamics of the stellar and gaseous components. In particular the discs of the Milky Way and of NGC 6946 are studied. The Milky Way is shown to be stable, whereas the inner parts of NGC 6946, a typical Sc galaxy from the Kennicutt (1989) sample, are dynamically unstable. The ensuing dynamical evolution of the composite disc is studied by numerical simulations. The evolution is so fierce that the stellar disc heats up dynamically on a short time scale to such a degree, which seems to contradict the morphological appearance of the galaxy. The star formation rate required to cool the disc dynamically is estimated. Even if the star formation rate in NGC 6946 is at present high enough to meet this requirement, it is argued that the discs of Sc galaxies cannot sustain such a high star formation rate for longer periods.

Key words: galaxies: kinematics and dynamics–galaxies: structure– galaxies: evolution– Galaxy: kinematics and dynamics– galaxies: individual: NGC 6946

1 INTRODUCTION

It has been long suspected that some Sc galaxies are so gas rich that their gaseous discs reach the threshold of dynamical instability (Quirk 1972). In an influential study Kennicutt (1989) has shown for a considerable sample of Sc galaxies by careful analysis of the distribution of atomic and molecular hydrogen in each galaxy that in the inner parts of the gaseous discs of the galaxies the Toomre (1964) stability condition is indeed violated. Kennicutt argues further that, since the galactocentric distances of threshold of dynamical instability coincide closely with the outer boundaries of the HII region discs of the galaxies, dynamical instabilities have led to the enhanced massive star formation rate in the inner parts of Sc galaxies. There are, however, counter examples in his sample, notably M33 and NGC 2403, which do not reach the threshold level. In addition Ferguson et al. (1994) have shown that HII regions can be also found in the outer – stable – parts of the discs of galaxies.

Even though it is intuitively plausible that dynamical instability leads eventually to enhanced star formation as observed in the Sc galaxies, we wish to point out that the existence of an unstable gas disc has grave consequences for the dynamics not only of the gaseous disc but for the stellar disc as well. For this purpose we examine in section 2 stability criteria for composite stellar and gas discs and derive a dispersion relation in order to estimate the time scale on which the instabilities develop. In section 3 we apply the stability criterion to the discs of the Milky Way and NGC 6946, a typical representative from the Kennicutt (1989) sample. The disc of NGC 6946 is shown to be dynamically unstable and we study its fierce dynamical evolution by numerical simulations in section 4. The implications of these simulations are discussed in the final section.

2 STABILITY CRITERION

The dynamical stability of the composite gaseous and stellar disc is examined, which gives a local stability criterion. Spiral instabilities and the bar instability, which are less violent, are not considered in this section. In previous stability studies of multicomponent galactic disc models the stellar disc has usually been described by Jeans equations (Biermann 1975, Jog & Solomon 1984a,b, Bertin & Romeo 1988, Elmegreen 1995, Jog 1996). Since the shape of the velocity distribution of the stars governs to some degree the stability of the stellar disc we prefer a full stellar–dynamical treatment.

It has become customary to describe the stellar velocity distribution by a Schwarzschild distribution. Wielen and Fuchs (1983), however, have pointed out that a velocity distribution with an exponential shape appears to be more realistic, at least for stars in the Milky Way, because the velocity distribution is made up of many generations of stars with velocity dispersions varying according to their age. First attempts to model such a distribution in the context of stability studies have been made by Morozov (1981).

In Fig. **??** a one–dimensional projection of a distribution function of the form

$$f_0 = \frac{3}{2\pi\sigma_U\sigma_V} \exp -\sqrt{3}\sqrt{\left(\frac{U}{\sigma_U}\right)^2 + \left(\frac{V}{\sigma_V}\right)^2} \qquad (1)$$

is shown in comparison with the observed velocity distribution of the

McCormick stars – a kinematically unbiased sample – in the Third Catalogue of Nearby Stars (Gliese and Jahreiß 1991). In equation (1) U and V are the radial and circumferential velocity components of the stars, respectively, and σ_U and σ_V are the corresponding second moments of the velocity distribution which we call loosely in the following velocity dispersions. The ratio of the velocity dispersions is given by the epicyclic ratio $\sigma_V/\sigma_U = -2B/\kappa$, with κ the epicyclic frequency and B Oort's constant. As can be seen in Fig. **??** there are a lot of low velocity stars, which is well modelled by the distribution function (1).

In his original study of the stability of stellar discs against axisymmetric perturbations Toomre (1964) assumed a Schwarzschild distribution for the stellar velocities. Graham (1965) and independently Toomre (cited in Graham (1965)) generalised the results to arbitrary distribution functions depending on the radial action integral $U^2 + \frac{\kappa^2}{4B^2}V^2$ (see also the appendix). The distribution function (1) is of this type. Graham (1965) did not consider explicitly the case of an exponential distribution. But Toomre has treated this case in his unpublished material, which he kindly made available to us. The main result is that in case of an exponential distribution function the standard Toomre stability parameter is slightly modified as

$$Q' = \frac{\kappa\sigma_U}{3.944 G\Sigma_0}, \tag{2}$$

where Σ_0 denotes the surface density of the disc and G is the constant of gravitation. The numerical factor 3.944 replaces a factor 3.36 in the case of a Schwarzschild distribution or a factor π in the case of an ideal gas. This indicates that a stellar disc with an exponential distribution is slightly more unstable than a disc with a Schwarzschild distribution, which has fewer low velocity stars.

We treat the interstellar gas as an isothermal gas, which will be sufficient to find the transition of the composite disc to gravitational instability (Cowie 1981). Graham (1967) has indicated how to extend his or Toomre's work to multicomponent disc models. Unfortunately, neither he nor Toomre have treated the case of an isothermal gas disc inbedded in a stellar disc with an exponential velocity distribution, which we consider here. In the appendix we discuss the stability of such a disc against axisymmetric perturbations adapting the analyses of Toomre and Graham to the present problem. The domain of neutrally stable perturbations in parameter space, i.e. the stability parameters of the gas and stellar discs, respectively, and the gas–to–stellar surface density ratio, indicates then which discs are dynamically stable. For the cases, where the stability condition is violated, we derive a dispersion relation, which allows the determination of the wave length and exponential rise time of the fastest growing perturbations.

3 STABILITY OF GALACTIC DISCS

The stability criterion developed in section 2 is applied to the discs of the Milky Way and NGC 6946.

3.1 Milky Way

Table 1 summarises the parameters of the Galactic disc which we adopt.

The resulting stability parameters are $Q_* = 2.4$ and $Q_g = 2.2$, respectively. On the other hand, using again the parameters given in Table 1, the curve of neutrally stable perturbations derived from equation (A19) may be calculated as shown in Fig. 2. The minimal value of the stability parameter Q_g required to stabilise the disc is $Q_{g,min} = 1.1$ implying that the Galactic disc is *stable* in the solar neighbourhood.

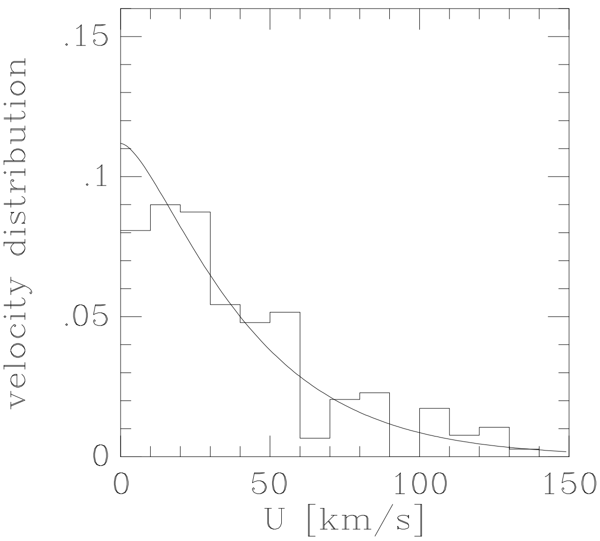

Figure 1. Distribution of – radial – U velocities of 317 McCormick stars in the solar neighbourhood. The observed distribution is approximated by the projection of an exponential distribution with a radial velocity dispersion of 48 km/s.

Table 1. Local Parameters of the Galactic Disc

κ:	36.6	km/s/kpc[a]
σ_U:	48	km/s[b]
$\Sigma_* + \Sigma_g$:	50	$\mathcal{M}_\odot$/pc^2[c]
Σ_g:	7.5	$\mathcal{M}_\odot$/pc^2[d]
c:	6	km/s

[a] IAU 1984 galactic constants and assuming a flat rotation curve
[b] from solar neighbourhood data (cf. Fig. **??**)
[c] Kuijken and Gilmore 1989a,b,c
[d] Dame 1993, multiplied by a factor of 1.4 to account for heavy elements

The stability of the inner and outer parts of the Galaxy may be analysed in the same way. The surface density and – squared – velocity dispersion of the stars can be extrapolated by an exponential law with a scale length of 4.4 kpc (Lewis & Freeman 1989). The surface density of the interstellar gas is taken from Dame (1993) and the turbulent velocity dispersion is assumed to be constant. The resulting stability parameters are listed in Table 2, implying *stability* of the Galactic disc against axisymmetric perturbations in the inner and outer parts as well. Similar conclusions were drawn by Elmegreen (1995) and Jog (1996) on the basis of their models.

Dynamical stability of the disc is consistent with the observation that there is no enhanced massive star formation such as in the Sc galaxies of the Kennicutt (1989) sample in the Milky Way. As was stressed previously by Ferguson et al. (1994) one finds, on the other hand, still star formation at moderate rates in galactic discs, which are dynamically stable in the sense discussed here. Apparently the complex structure of the interstellar medium, which is not modelled here, still allows localised Jeans collapse. It appears that

Table 2. Stability Parameters of the Galactic Disc

	R = 4.5 kpc	$R_\odot$ = 8.5 kpc	R = 12 kpc
Q_*	2.9	2.4	2.6
Q_g	2.1	2.2	1.7
$Q_{g,min}$	1.1	1.1	1.3

Table 3. Stability Parameters during the Evolution of the Galactic Disc (R = $R_\odot$)

t_{ep}	Q_*	Q_g	$Q_{g,min}$
10^{10} yrs	2.4	2.2	1.1
$5 \cdot 10^9$ yrs	1.7	2.2	1.4
$2.5 \cdot 10^9$ yrs	1.2	2.2	5.5
$1.25 \cdot 10^9$ yrs	0.9	2.2	> 20

dynamical instability is not a necessary condition for star formation but greatly enhances the star formation rate.

The radial scale length of the Milky Way is not well constrained and, as can be seen in the synopsis by Sackett (1997), may be for the main populations of the stars in the Galactic disc as small as 2.5 kpc. This would make the inner parts of the Galactic stellar disc more unstable. However, the effect is not significant. Assuming that the Galactic disc has a constant vertical scale height as function of galactocentric radius one would expect the – squared – velocity dispersion of the stars to have the same radial gradient as the surface density of the stars, which implies $Q_\star = 2.1$ at $R = 4.5$ kpc.

The finite thickness of the disc, which has not been considered here explicitly, has a further stabilising effect on the disc.

It is interesting to see whether due to the age dependence of the velocity dispersion of the stars the Galactic disc was dynamically unstable at earlier stages of its evolution. The velocity dispersion varies today as $\sigma_U^2 \propto \tau$, with τ denoting the ages of the stars (Wielen 1977). So, assuming a constant star formation rate, it is to be expected that at an earlier epoch t_{ep} the velocity dispersion of all stars which were born until then was

$$\overline{\sigma_U^2} = \frac{1}{t_{ep}} \int_0^{t_{ep}} \sigma_U^2(\tau)\, d\tau = \frac{1}{2}\, \sigma_U^2\ (\tau = t_{ep}). \tag{3}$$

Table 3 shows the evolution of the stability parameters of the Galactic disc due to this effect assuming that the other parameters of the disc were the same as today.

Apparently, this indicates instability of the disc during early stages of galactic evolution. This would be in line with Kennicutt et al.'s (1994) argument that early type spirals had apparently an initial burst of star formation. However, we believe, for reasons explained in the next sections, phases of dynamical instability extending over more than 10^9 years unlikely. Probably, the surface density of the disc was then lower than today and a lot of material might have been accreted later onto the disc (Gunn 1982). If, for example, the surface density was only one half of its present value and the gas density equal to the stellar density, the stability parameters were Q_* = 4, Q_g = 1.3, whereas the minimal value of the parameter required to stabilise the disc was $Q_{g,min}$ = 1.2.

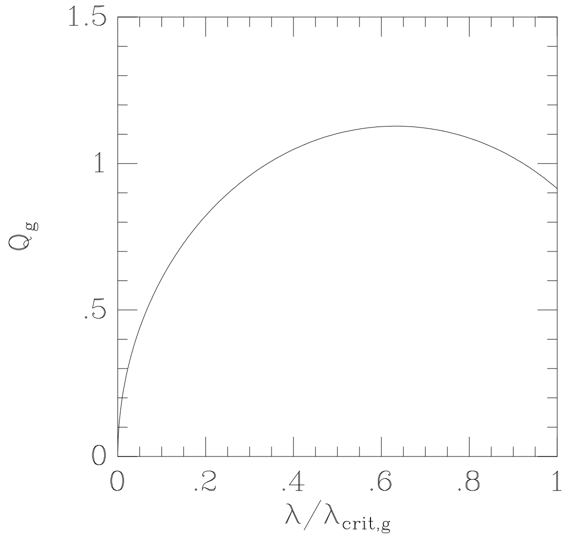

Figure 2. Curve of neutrally stable axisymmetric perturbations of a gas disc embedded in a stellar disc. Local parameters of the Galactic disc have been used.

3.2 NGC 6946

NGC 6946 is a typical representative of the Sc galaxies studied by Kennicutt (1989). The absolute mass as well as the radial distribution of atomic and molecular hydrogen in NGC 6946 have been determined by Tacconi & Young (1986). The rotation curve has been observed by Carignan et al. (1990), who have also constructed a mass model comprising a disc and dark halo component. Unfortunately, the stellar velocity dispersion is not known, but one may derive an estimate from the vertical scale height of the disc. Friese et al. (1995) have shown in a statistical flattening analysis of faint spiral galaxies in the ESO–Uppsala catalogue that the typical intrinsic ratio of vertical to radial scale lengths of Scd galaxies is of the order of z_0/h = 0.25 implying z_0 = 1.4 kpc. This may be used with the vertical hydrostatic equilibrium condition to derive an estimate of the vertical velocity dispersion of the stars,

$$\sigma_W^2 = \mathcal{F}\pi G \Sigma_0 z_0. \tag{4}$$

$\mathcal{F}$ is a form factor which is equal to 1 for an isothermal disc with a vertical $sech^2$ density profile and about 2 for the Milky Way disc (Friese et al. 1995). The same value is also adopted for the disc of NGC 6946. The radial velocity dispersion is estimated by using the same axial ratio of velocity ellipsoid as in the Milky Way, $\sigma_U^2/\sigma_W^2 = 1 + \kappa^2/4B^2$. In columns 5, 8, and 9 of Tab. 4 the radial variations of the stability parameters, which have been calculated including finite thickness corrections, are shown. We note that the stability parameter of the stellar disc determined in this way, $Q_\star \approx 2$, is in good agreement with values determined by Bottema (1993) for galaxies for which kinematic data are available.

There is a distinct drop of the stability parameter of the gas disc Q_g inside R = 16 kpc, which, as was pointed out by Kennicutt (1989), is correlated with the outer boundary of the HII region disc

Table 4. Radial Variation of Stability Parameters of the Disc of NGC 6946 (d = 10 Mpc)

R	Σ_*	κ	σ_U	Q_*	Σ_g^a	c	Q_g	$Q_{g,min}^b$	$-\Im(\omega)/\kappa$	λ_{max}
5	133	50	114	2.5	38	6	0.6	0.9	0.9	0.7
10	55	23	89	2.2	21	6	0.5	1.1	1.4	1.0
12	38	20	76	2.3	17	6	0.5	1.1	1.4	1.2
14	27	17	66	2.4	14	6	0.5	1.1	1.4	1.5
16	19	15	52	2.3	6	6	1.0	1.0	—	—
18	13	13	44	2.6	5	6	1.1	1.0	—	—
20	9	12	39	3.0	5	6	1.0	1.0	—	—
kpc	$\frac{M_\odot}{pc^2}$	$\frac{km/s}{kpc}$	km/s		$\frac{M_\odot}{pc^2}$	km/s				kpc

[a] Molecular hydrogen surface densities have been rescaled to the CO density – to – H_2 column density conversion factor given by Dame (1993). Atomic and molecular hydrogen surface densities have been multiplied by a factor 1.4 to account for heavy elements.

[b] Corrected for the finite thickness of the discs, assuming $z_{0,\star} = 1.4$ kpc and $z_{0,g} = 50$ pc.

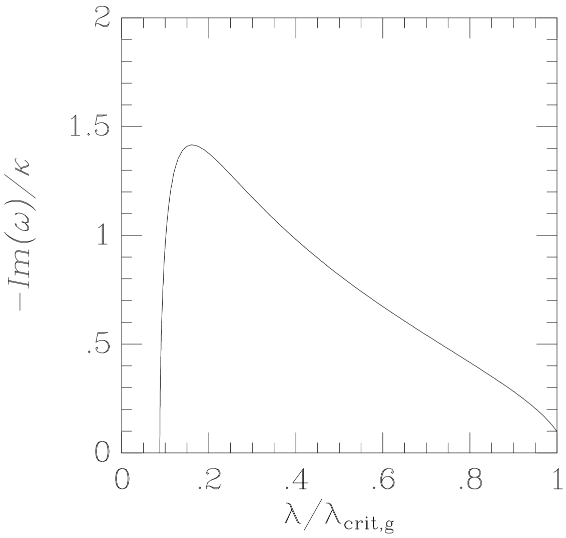

Figure 3. Dispersion relation of exponentially growing axisymmetric perturbations of a composite gas and stellar disc. Parameters of the disc of NGC 6946 at galactocentric distance R = 10 kpc have been used.

of NGC 6946. As can be seen from column 9 of Table 4 the stability criterion as presented here indicates in accordance with Kennicutt's conclusions that the inner parts of the disc of NGC 6946 are actually *dynamically unstable*. This is mainly due to the large gas content of NGC 6949, which is quite typical for late type spiral galaxies (Kennicutt et al. 1994). The exponential growth rates as well as the most unstable wavelengths, which may be calculated from the combined dispersion relation (A19) are illustrated in Fig. **??** and are given in columns 10 and 11 of Table 4. The e-folding rise time of the instability is rather short, only $3{\cdot}10^7$ yrs at R = 10 kpc corresponding to 0.1 epicyclic periods. The unstable wavelengths are short compared to the radial extent of the galactic disc.

Using equations (**??**) and (**??**) one can show that, even if the instability grows at the same rate in the gaseous and the stellar discs, the amplitude of the disturbance of the stellar disc is much smaller than in the gaseous disc. For the parameters adopted in Table 4 the relative star–to–gas density contrast is typically of the order of 10^{-3}

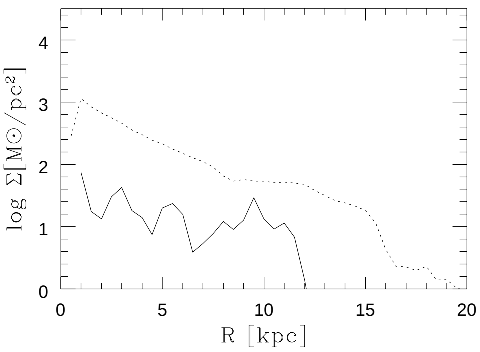

Figure 4. Surface density distribution of the stellar (dotted line) and the gas (solid line) at time step t=1.2 $\cdot 10^7$ yrs of the simulation.

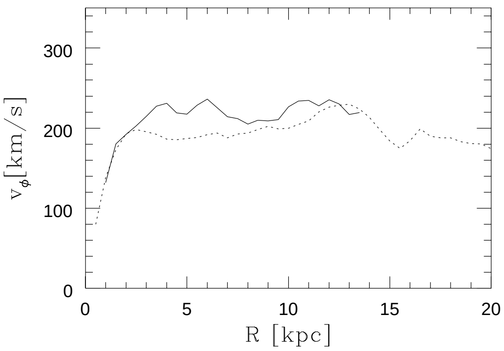

Figure 5. Rotation curve of the stars (dotted line) and the gas (solid line) at time step t=$1.2 \cdot 10^7$ yrs of the simulation.

so that, even when the perturbations of the gas disc become highly non–linear, the stellar disc is only mildly perturbed.

4 NUMERICAL SIMULATIONS

The onset of the instability into the non–linear regime can be followed by numerical simulations. Simulations of a dynamically unstable gas disc inbedded in a stellar disc were first carried out by

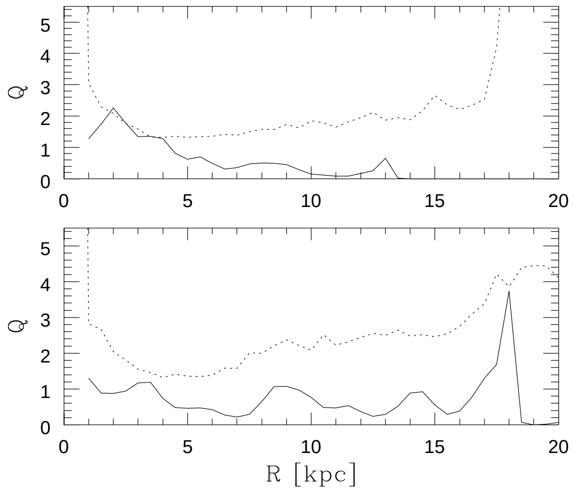

Figure 6. Stability parameters of the stellar disc (dotted lines) and the gas disc (solid lines) at time steps t=$6{\cdot}10^7$ yrs (upper panel) and t=$18\cdot 10^7$ yrs (lower panel) of the simulation.

Carlberg & Freedman (1985). We have rerun similar simulations in order to analyse them in the present context. For this purpose we have used a code developed by F. Combes and collaborators (cf. Casoli and Combes 1982, Combes and Gérin 1985 for the full details). The code implements a two–dimensional stellar disc ($N_{star} = 38\,000$) into which interstellar gas clouds are embedded ($N_{clouds} \le 38\,000$). This composite disc is surrounded by a rigid bulge and dark halo potential. The gravitational potential of the disc is calculated by a standard particle–mesh scheme and the orbits of the stars and the clouds are integrated numerically. The stars interact only by – softened – gravitational forces, while the gas clouds may interact inelastically. This is simulated in an elaborate cloud–in–cell scheme, which describes the coalescence and fragmentation of the gas clouds. In this way a mass spectrum of the gas clouds is established. The various parameters of the collision scheme have been adjusted by Combes & Gérin (1985) in such a way that the mass spectrum of the gas clouds resembles the mass spectrum of molecular cloud complexes in the Milky Way. A finite lifetime of $4{\cdot}10^7$ years due to massive star formation is assumed for the giant molecular clouds (GMC) at the high end of the mass spectrum, $M_{\rm GMC} > 2\cdot 10^5\,{\rm M}_\odot$. After that the clouds are disrupted into small fragments expanding initially isotropically at relative speeds of 10 km/s.

The softening length of the gravitational forces is 500 pc. Following Romeo (1994) we have adjusted it in this way, because on one hand it is still considerably smaller than the critical wavelengths of the interstellar gas and stellar discs, respectively (see below). On the other hand, it is of the same order of magnitude of the expected vertical scale height of the Galactic disc, so that the stabilising effect of a finite thickness of the disc is taken into account to some extent in our two-dimensional simulations.

The stars are initially distributed according to a Toomre disc (Toomre 1963),

$$\Sigma_d(R) = \frac{M_d \cdot d}{2\pi} \cdot (R^2 + d^2)^{-3/2}\,, \tag{5}$$

where M_d denotes the mass of the disc and d is the radial scale length. The gas clouds are distributed in an exponential disc. The halo and the bulge are modelled by Plummer spheres. The parameters which we have adopted for the various components of the model are summarised in Table **??**. The begin of the simulation is

Table 5. Parameter adopted for the simulation

stellar disc mass M_d	7.7	$10^{10}\,{\rm M}_\odot$
scale length d	4	kpc
truncation radius	32	kpc
gas disc mass M_g	0.6	$10^{10}\,{\rm M}_\odot$
exponential scale length	16	kpc
dark–halo mass M_h	10	$10^{10}\,{\rm M}_\odot$
core radius	24	kpc
bulge mass M_b	2.4	$10^{10}\,{\rm M}_\odot$
scale length	1.5	kpc
mass ratio $(M_d + M_g)/M_{\rm tot}$	0.40	

illustrated in Figs. **??** to **??**, where the surface densities, the rotation curve, and the stability parameters are shown. Fig. **??** shows that the the combined disc of stars and interstellar gas clouds is *dynamically unstable* and resembles in that the inner parts of the Sc galaxies in the sample of Kennicutt (1989). The next steps of the evolution of the disc are shown in Fig. **??** at multiple intervals of Δt = $1.2{\cdot}10^7$ years which correspond to about 0.1 epicyclic periods. As expected ring–like density perturbations appear immediately in the gas disc. The wave lengths of these perturbations are of the order of the critical wave length of the gas disc, $\lambda_{crit,g}$ about 2 kpc at R = 10 kpc. The rings fragment into lumps with masses in the range of 10^4 to 10^7 ${\rm M}_\odot$. These agglomerates are so heavy that the stellar disc responds to them by induced, 'swing–amplified' spiral structures. According to the 'X=2' criterion (Toomre 1981) swing–amplification is most effective for spirals with a number of

$$m = \frac{1}{2} \cdot \frac{2\pi R}{\lambda_{crit}} \tag{6}$$

spiral arms. Since λ_{crit} = 10 kpc, the expected number of spiral arms is about m = 3, which is typically seen in our simulations (see for instance time step $14.4\cdot 10^7$). The potential troughs of the stellar disc, on the other hand, begin to trap much of the interstellar gas and during their further evolution both the stellar disc as well as the gas disc undergo rather synchronous, repetitive cycles of swing amplified spiral perturbations.

After $5{\cdot}10^8$ years the stellar disc gets heated up dynamically by the spiral activity so much that hardly any non–axisymmetric structure is longer possible in the disc.

In the gaseous disc, however, there is still a lot of spiral activity. Since the stellar disc has become inactive the critical wavelengths of the spiral structures are much smaller and the number of expected spiral has risen to about 20 in accordance with flocculent appearance of the disc. The same phenomenon was described by Carlberg & Freedman (1985)

5 DISCUSSION

The numerical simulations show that the discs of Sc galaxies like NGC 6946 are in a highly peculiar dynamical state. The reaction of the stellar discs to dynamically unstable gas discs is so fierce that they become dynamically hot within less than 10^9 years. On the other hand, Toomre (1990) has argued emphatically that Sc galaxies must have stellar discs, which are dynamically active, because otherwise the morphological appearance would be quite different

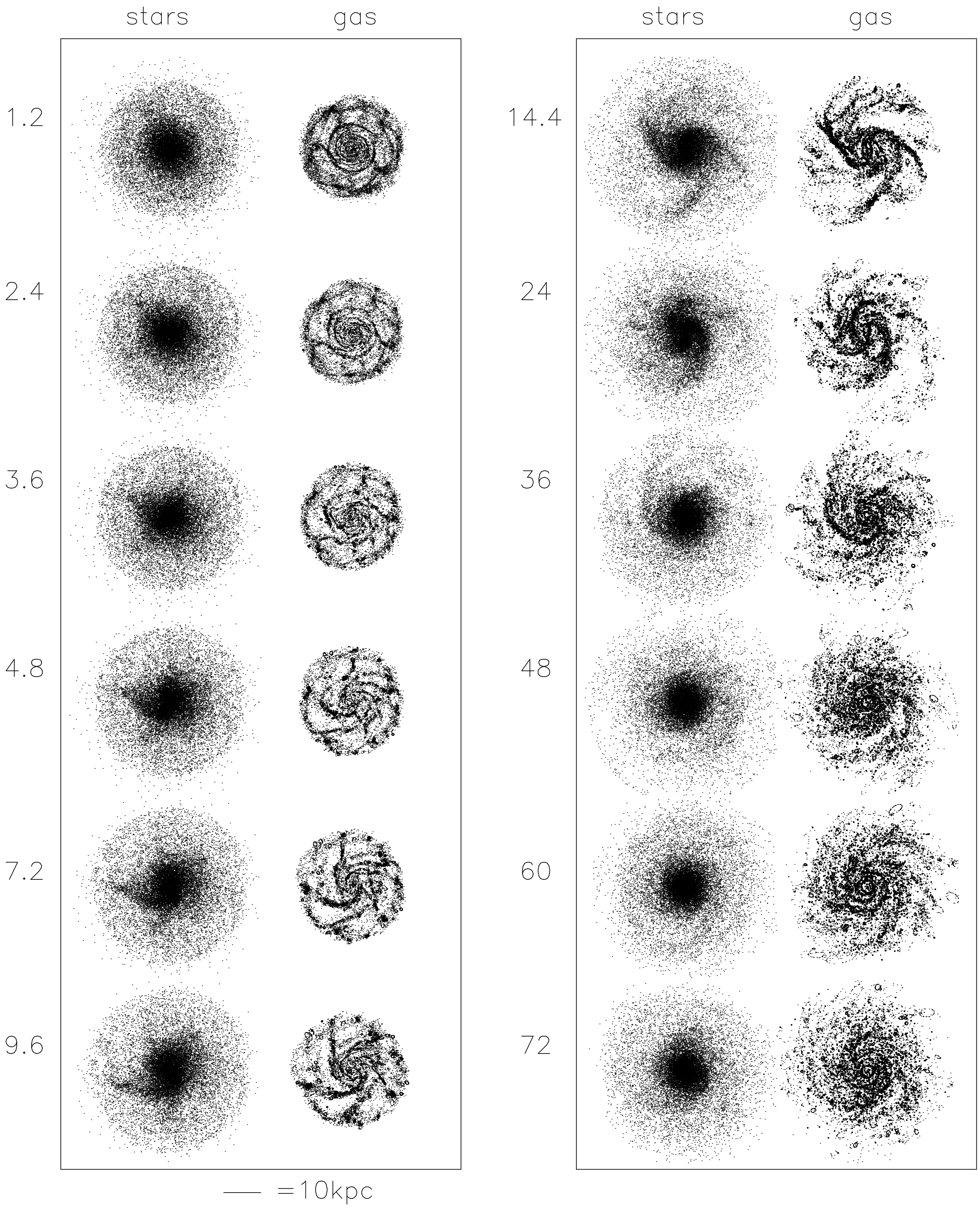

Figure 7. Dynamical evolution of the stellar and gaseous discs. On the left of each panel 19 000 out of 38 000 stars and on the right hand side of each panel19 000 of about 38 000 interstellar gas clouds are plotted at consecutive time intervals. The time is indicated on the left in units of 10^7 years. The spatial size is indicated at the bottom by a bar of 10 kpc length.

from what is actually observed. This can be clearly seen in Fig. 7, when one compares the frames corresponding of say $1.44 \cdot 10^8$ years and $4.8 \cdot 10^8$ years with an optical image of the galaxy (Sandage & Bedke, 1988). Thus the stellar discs in Sc galaxies must be effectively cooled dynamically by newly formed stars on low velocity dispersion orbits. The star formation rate required to keep the stellar discs in a steady state can be roughly estimated as follows. After $5 \cdot 10^8$ years the stellar disc has heated up so much that it tunes out of spiral activity and the – squared – stability parameter has risen to

$$Q_\star^2 = \frac{\kappa^2 \sigma_\star^2}{(3.36 G \Sigma_\star)^2} \approx 6. \tag{7}$$

Newly formed stars will lower this to

$$Q_\star'^2 = \frac{\kappa^2 (\Sigma_\star \sigma_\star^2 + \delta_\star \Sigma_\star \sigma_{\star,i}^2)}{(3.36G)^2 (1+\delta_\star)^3 \Sigma_\star^3}, \tag{8}$$

where we assume that the surface density of the stellar disc has risen due to star formation to $\Sigma_\star' = \Sigma_\star(1+\delta_\star)$ and that the – squared – velocity dispersion of the mixture of older and younger stars can be approximately estimated by an mass weighted average of the velocity dispersions of the components. Combining equation (**??**) and (**??**) gives

$$(1+\delta_\star)^3 = \frac{Q_\star^2}{Q_\star'^2}(1+\delta_\star \frac{{\sigma_{\star,i}}^2}{\sigma_\star^2}) \simeq \frac{Q_\star^2}{Q_\star'^2}, \tag{9}$$

if the velocity dispersion of the newly born stars, $\sigma_{\star,i}$, is much smaller than the average velocity dispersion of the stars. In our simulations the stability parameter $Q_\star$ rose from about initially 2 to 2.5 at $t = 4.8 \cdot 10^5$ yrs. According to equation (9) about 40 % of the mass of the stellar disc is required per 10^9 years in the form of newly born stars to keep the stellar disc in a steady dynamical state. Obviously this material must be provided by the gaseous disc. It is interesting to compare this estimate of the gas consumption rate with the actual gas consumption rate in NGC 6946. This can be deduced from the star formation rate which in turn can be quantitatively estimated from the H_α flux using the relation of Kennicutt (1983). In Table 6 we give the radial distribution of the star formation rate determined from the extinction corrected H_α surface emissivity of NGC 6946 (Devereux & Young 1993). The surface densities of the gaseous and stellar discs shown in Table 6 are taken form Table 4. In the last column of Table 6 we give the relative increase of the stellar surface density per 10^9 years, $\delta_\star$, estimated from the previous parameters. As can be seen from Table 6 the actually observed star formation rate and thus the disc mass increase is at present, if compared with the theoretical estimate found above, high enough to keep the stellar disc dynamically cool. In this aspect the disc seems to be self regulating. However, the gas consumption rate is very large. As is shown in Table 6 star formation seems at present to consume nearly the entire gas disc within 10^9 yrs. This implies either a very high gas accretion rate, which at this magnitude seems unlikely to us, or that the discs of the Sc galaxies of the NGC 6946 type will soon switch over to more quiescent dynamical states like in M33 or NGC 2403. In these galaxies the gaseous discs do not reach the threshold of dynamical instability. But even if the stability parameter Q_g is slightly larger than 1, this would mean still quite a lot of spiral activity in the disc. This might account for the still comparatively high star formation rates observed in these galaxies (Kennicutt et al. 1994), but leads also to considerable dynamical heating (Carlberg & Sellwood 1984). The same comment applies to the low surface brightness galaxies discussed by Mihos et al. (1997).

Table 6. Radial variation of the stellar disc mass increase per 10^9 yrs in NGC 6946

R	SFR	$\Sigma_\star$	Σ_{gas}	$\delta_\star$
0	285	325	306	0.9
1.1	180	267	143	0.7
1.2	71	219	95	0.3
4.4	57	148	53	0.4
6.6	28	100	36	0.3
8.8	11	67	27	0.2
11	9	46	15	0.2
13.2	4	31	11	0.1
kpc	$\frac{M_\odot}{pc^2}/Gyr$	$\frac{M_\odot}{pc^2}$	$\frac{M_\odot}{pc^2}$	

A further interesting example is the Sb galaxy NGC 7331. Its radial distribution of atomic and molecular hydrogen is given by Young & Scoville (1982). After rescaling the molecular hydrogen densities to a modern CO intensity–to–H_2 column density ratio (Dame 1993) it can be shown that the gaseous disc of NGC 7331 does not reach the threshold of dynamical instability, $Q_g \tilde{>} 1$. In accordance with this result the star formation rate seems to be in NGC 7331 much lower than in NGC 6946. Applying again the Kennicutt (1983) relation to the observed integral H_α flux (Young et al. 1996) after correcting this for internal extinction in the way suggested by Devereux & Young (1993) we estimate a global star formation rate of 7 $M_\odot$/yr. This is about half the value found for NGC 6946. Furthermore, the gaseous and, in particular, the stellar disc (Broeils 1995) are more massive than in NGC 6946 so that the relative disc mass increase is only moderate.

Finally we note that the adopted distances to the galaxies affect the dynamical stability condition of their gas discs (cf. Zasov & Bizyaev 1996). The surface densities and amplitudes of the rotation curves are distance independent. The epicyclic frequency scales inverse proportional to the distance, while the estimate of the turbulent velocity dispersion of the gas used throughout this study is distance independent. Thus assuming half of the presently adopted distance to NGC 6946, as suggested by de Vaucouleurs (1979), raises the stability parameter of the gas disc close to the threshold of dynamical instability.

6 CONCLUSIONS

We have formulated a local stability criterion of Galactic discs against axisymmetric density disturbances modelling the different dynamics of the stellar and gaseous components. The disc of the Milky Way is shown to be dynamically stable at all Galactic radii and probably over most of its past history. The inner parts of the disc of NGC 6949, a typical Sc galaxy from the Kennicutt (1989) sample, are found to be dynamically unstable. We have followed the ensuing dynamical evolution of the disc by numerical simulations. These show that such unstable discs evolve very rapidly. In order to stay in its present state the stellar disc would have to be effectively cooled by star formation. This seems to be actually observed in NGC 6946. However, the gas disc would have to be replenished by heavy accretion of gas, amounting to several times the present day gas disc mass during a Hubble time.

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APPENDIX A1:

The stability of the self-gravitating composite gas and stellar disc is examined. Both disc components are approximated as infinitesimal thin sheets. In order to test the stability of the disc it is subjected to density perturbations of the form

$$\exp i(\omega t + kR)\,, \tag{A1}$$

where ω and k denote the –complex– frequency and radial wave number, respectively. Since the most unstable wavelengths $\lambda = 2\pi/k$ turn out to be small compared to the radial extent of the disc it is sufficient to study the stability of the disc in a localised theory (Toomre 1964). Considering now a strip of the galactic disc, radial variations of the surface density and of the velocity distribution of the stars and the interstellar gas can be neglected.

A1.1 Stellar disc

In order to describe the dynamics of the stars in the circular strip around the galactic centre we consider the collision-less Boltzmann equation,

$$\frac{\partial f}{\partial t} + [f, H] = 0, \tag{A2}$$

where f denotes the distribution function of the stars in phase space and H is the Hamiltonian. The distribution function of the unperturbed disc, f_0, is chosen according to equation (1) and normalised to the surface density Σ_0. Radial variations of f_0 are neglected. The – axisymmetric – disc response f_1 to a small axisymmetric potential perturbation, $\delta\Phi = \Phi_k \exp ikR$, is calculated from the linearised Boltzmann equation

$$\frac{\partial f_1}{\partial t} + [f_1, H_0] + [f_0, \delta\Phi] = 0. \tag{A3}$$

In the following the plane stellar orbits are described according to the epicyclic approximation. The Hamiltonian H_0 is then given by

$$H_0 = \frac{1}{2}\dot{R}^2 + \frac{1}{2}R_0^2(\dot{\theta} - \Omega_0)^2 - 2A\Omega_0(R - R_0)^2, \qquad \text{or alternatively} \qquad H_0 = \frac{J_1}{2} + \frac{A}{2B}(J_2 - \Omega_0 R_0)^2 - \frac{1}{2}\Omega_0^2 R_0^2, \tag{A4}$$

where A and B denote Oort's constants. This leads to the equations of motion

$$R - R_0 = \frac{J_2 - \Omega_0 R_0}{-2B} + \sqrt{\frac{2J_1}{\kappa}} \sin w_1, \qquad R_0(\theta - \Omega_0 t) = w_2 - \frac{\sqrt{2\kappa J_1}}{2B} \cos w_1, \tag{A5}$$

$$U = \dot{R} = \sqrt{2\kappa J_1} \cos w_1, \qquad V = R_0(\dot{\theta} - \Omega_0) + 2A(R - R_0) = 2B\sqrt{\frac{2J_1}{\kappa}} \sin w_1, \tag{A6}$$

where R, θ denote polar coordinates. R_0 is the mean galactocentric radius of the strip of the disc under consideration. Ω_0 is the mean angular velocity of the stars in the strip around the galactic centre. J_1 and J_2 are integrals of motion, $J_1 = (U^2 + \frac{\kappa^2}{4B^2}V^2)^{1/2}$ the radial action integral and $R_0\,(J_2 - \Omega_0 R_0)$ the angular momentum of a star referred to the mean radius of the strip. w_1 and w_2 denote the conjugate variables derived from the Hamiltonian H_0,

$$w_1 = \kappa t, \qquad w_2 = \frac{A}{B}(J_2 - \Omega_0 R_0)t. \tag{A7}$$

The Poisson brackets in equation (A3) are then given by

$$[f_1, H_0] = \kappa \frac{\partial f_1}{\partial w_1}, \qquad [f_0, \delta\Phi] = -\frac{\partial f_0}{\partial J_1} \cdot \frac{\partial \delta\Phi}{\partial w_1} = \frac{\sqrt{3}}{\sigma_U}\delta\Phi ik \cos w_1. \tag{A8}$$

Since the disc response is axisymmetric no $\partial f_1/\partial w_2$ derivatives appear in equation (**??**). This leads to the final form of the linearised Boltzmann equation

$$\frac{\partial f_1}{\partial t} + \kappa\frac{\partial f_1}{\partial w_1} = -\Sigma_0 \frac{\kappa}{-2B}\frac{3\sqrt{3}}{2\pi\sigma_U^3} \exp - \left(\frac{\sqrt{3}}{\sigma_U}\sqrt{2\kappa J_1}\right) ik\Phi_k \cos w_1 \exp ik \left(\frac{J_2 - \Omega_0 R_0}{-2B} + \sqrt{\frac{2J_1}{\kappa}} \sin w_1\right). \tag{A9}$$

Assuming a time dependence according to equation (A1),

$$f_1, \Phi_k \propto \exp i\omega t, \tag{A10}$$

equation (**??**) can solved by combining solutions of the homogeneous part of the equation and a particular solution of the inhomogeneous equation found by 'variation of the constant', giving finally

$$f_1 = -\frac{f_0}{\sqrt{2\kappa J_1}} e^{i\omega t}\frac{\sqrt{3}}{\sigma_U}\Phi_k e^{ik(R-R_0)} \cdot \left\{1 - \frac{\omega/\kappa}{2\sin(\pi\omega/\kappa)}\int_{-\pi}^{\pi} dw_1' \exp i\left[\frac{\omega}{\kappa}w_1' - \sqrt{\frac{2J_1}{\kappa}}k\left(\sin w_1 + \sin(w_1 + w_1')\right)\right]\right\}. \tag{A11}$$

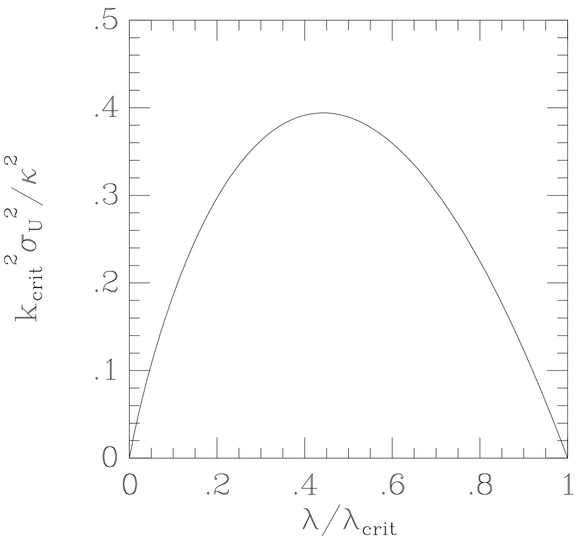

Figure A1. Curve of neutrally stable axisymmetric perturbations of a stellar disc with an exponential velocity distribution. Unstable solutions lie below the curve.

In order to have a distribution function f_1, which is uniquely defined in velocity space, the integration constant of the solution (**??**) has been chosen in such a way that f_1 is periodic with respect to the angular variable w_1 (cf. Toomre 1964). The density response of the disc to the potential perturbation is found by integrating the distribution function f_1 over velocity space,

$$\begin{aligned}\Sigma_{1,k} &= -\int_0^\infty dJ_1 \int_0^{2\pi} dw_1 \Sigma_0 \frac{3\sqrt{3}\kappa}{2\pi\sigma_U^3} \exp - \left(\frac{\sqrt{3}}{\sigma_U}\sqrt{2\kappa J_1}\right) \frac{1}{\sqrt{2\kappa J_1}} \Phi_k e^{ik(R-R_0)} \Bigg\{ 1 - \frac{\omega/\kappa}{2\sin\left(\pi\omega/\kappa\right)} \\ &\quad \cdot \int_{-\pi}^{\pi} dw_1' \exp i \left[\frac{\omega}{\kappa} w_1' - \sqrt{\frac{2J_1}{\kappa}} k \left(\sin w_1 + \sin\left(w_1 + w_1'\right)\right)\right]\Bigg\}. \qquad \text{(A12)}\end{aligned}$$

The surface density perturbations are assumed to be self consistent. Therefore the density and potential perturbations have to satisfy the Poisson equation,

$$\Delta\delta\Phi = 4\pi G\Sigma_1\delta(z), \qquad \text{(A13)}$$

where $\delta(z)$ denotes a delta function with respect to the vertical spatial coordinate z. If the wave numbers $|k|$ are sufficiently large, $|k|R \gg 1$, the Laplace operator in equation (**??**) has nearly cartesian form and one obtains

$$\delta\Phi_k \propto e^{ik(R-R_0)-k|z|} \qquad \text{and} \qquad \Phi_k e^{ik(R-R_0)} = -\frac{\Sigma_{1,k}}{2\pi G|k|}. \qquad \text{(A14)}$$

Thus, the amplitude of the density perturbations, $\Sigma_{1,k}$, cancels out of equation (**??**). Finally one obtains after evaluating the quadratures with respect to J_1 and w_1 the dispersion relation.

$$1 = \frac{|\lambda|}{\lambda_{\rm crit}} \frac{3}{\xi} \left\{ 1 - \frac{\omega/\kappa}{2\sin(\pi\omega/\kappa)} \int_{-\pi}^{+\pi} dw_1' \frac{\exp(i\frac{\omega}{\kappa}w_1')}{\sqrt{1 + \frac{4}{3}\xi(\frac{\lambda_{\rm crit}}{\lambda})^2\cos^2(\frac{w_1'}{2})}} \right\}, \qquad \text{(A15)}$$

where $\lambda_{\rm crit} = 4\pi^2 G\Sigma_0/\kappa^2$ is the critical wavelength and $\xi = 4\pi^2\sigma_U^2/\lambda_{\rm crit}^2\kappa^2$. For $\omega = 0$ equation (A15) gives the curve of neutrally stable perturbations in the (λ, ξ) parameter space as shown in Fig. **??**. Exponentially unstable perturbations lie below the curve, whereas oscillatory solutions lie above. As can be seen from Fig. **??** discs with $\xi \geq 0.394$ are stable at all wavelengths. This corresponds to the Toomre parameter given equation (2).

In case of a three–dimensional disc of finite thickness the vertical oscillations of the stars have to be taken into account. In the epicyclic approximation, however, the vertical oscillations separate from the planar motions and the energy associated with the vertical oscillations has simply to be added to the Hamiltonian (A4). Vandervoort (1970a,b) has developed methods to solve the Boltzmann and Poisson equations for such a system. A rough estimate of the finite thickness effect on the stability of the disc can be found, however, according to Toomre (1964). It can be shown that potential perturbations of a disc with an effective scale height z_0 are reduced by a factor of

$$\frac{1-\exp(-kz_0)}{kz_0} \qquad \text{(A16)}$$

with respect to the potential perturbations of an infinitesimal thin disc with the same surface density. In order to correct the dispersion relation for this effect, the right hand side of equation (A15) has to be multiplied by the reduction factor (A16).

A1.2 Gaseous disc

The gaseous component of the galactic disc is modelled by an isothermal gas. As is well known (Binney & Tremaine 1987), the density response of the gaseous disc is given by

$$\Sigma_{g1,k} = \Sigma_{g0}\Phi_k \frac{k^2 e^{ik(R-R_0)}}{\omega^2 - c^2k^2 + 4\Omega_0 B}, \tag{A17}$$

where Σ_{g0} denotes the unperturbed surface density of the gaseous disc and c is the turbulent velocity dispersion.
Inserting into equation (A17) the analogue to equation (A14) leads to the dispersion relation of axisymmetric perturbations of a gas disc,

$$1 = \frac{\lambda_{\rm crit}}{\mid\lambda\mid}\, \frac{1}{1-(\frac{\omega}{\kappa})^2 + \frac{Q^2}{4}\,(\frac{\lambda_{crit}}{\lambda})^2}, \qquad \text{with} \quad Q = \frac{c\kappa}{\pi G \Sigma_0}\,. \tag{A18}$$

The effect of the finite thickness of the disc can be taken into account analogous to equation (A15) by multiplication with the reduction factor (A16).

A1.3 Stellar and gas disc combined

In the case of a two–component disc the density perturbation in equation (**??**) refers to the total surface density of the disc response. Thus, the density response of the gaseous disc (A17) has to be added to the density response of the stellar disc (A12) and the sum then inserted into equation (A14). The resulting dispersion relation has the form

$$1 \quad = \quad \frac{\mid\lambda\mid}{\lambda_{crit*}}\frac{3}{\xi_*}\left\{1 - \frac{\omega/\kappa}{2\sin\left(\pi\omega/\kappa\right)}\int_{-\pi}^{+\pi} dw_1' \,\frac{\exp(i\frac{\omega}{\kappa}w_1')}{\sqrt{1+\frac{4}{3}\xi_*(\frac{\lambda_{\rm crit*}}{\lambda})^2\cos^2(\frac{w_1'}{2})}}\right\} + \frac{\lambda_{crit,g}}{\mid\lambda\mid}\;\frac{1}{1-(\frac{\omega}{\kappa})^2+\frac{Q_g^2}{4}\,(\frac{\lambda_{crit,g}}{\lambda})^2}\,. \tag{A19}$$

where the parameters λ_{crit*}, ξ_* and $\lambda_{crit,g}$, Q_g are defined as in sections A1 and A2 for the stellar or the gaseous discs, respectively. Setting $\omega = 0$ gives the curve of neutral stable perturbations.