STATISTICAL ASPECTS OF STELLAR ASTROMETRY AND THEIR IMPLICATIONS FOR HIGH-PRECISION MEASUREMENTS

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ABSTRACT

Astrometric binaries represent a challenge for high-precision astrometry. Since astrometric binaries are frequent, their non-linear orbital motions (of the photo-center of the binary with respect to the center-of-mass) may be considered as 'cosmic errors' in the positions and proper motions of an ensemble of stars, if one assumes (wrongly) linear motions for all the stars. We outline a coherent scheme for 'statistical astrometry' which takes the statistical effect of astrometric binaries on astrometric procedures into account. The basic tools are correlation functions between the orbital displacements in position and velocity of a binary. We estimate the 'cosmic errors' in position and proper motion, and find that they are much larger than the measuring errors of a mission like GAIA. We discuss some implications of the astrometric-binary nature of many of the stars on the strategy of a mission and on the applications of the results. We emphasize that the data reduction procedure should include the accurate modelling of a possible orbital motion of all the stars. Then, for example, the high measuring accuracy of the parallaxes would not be lowered for most of the stars.

Keywords: astrometric binaries; high-precision astrometry; statistical astrometry; HIPPARCOS; GAIA.

1. THE STATISTICAL NATURE OF HIGH-PRECISION ASTROMETRY

High-precision astrometry of stars is severely affected by the orbital motions of undetected or unmodelled astrometric binaries. This effect introduces a strong statistical aspect into astrometry as soon as the accuracy of the measurements gets very high.

In classical astrometry, the basic assumption is that most of the stars move linearly in time on a straight line. Astrometric binaries are handled at best individually as rare exceptions. The assumption of linear motion implies that the proper motion is constant in time (except for some higher-order effects) and that the position of a star can be accurately predicted even for the distant past or future, if the position and proper motion of the star is accurately known at one epoch of time.

This basic assumption of linear motion of most of the stars breaks down in high-precision astrometry. The motion of the measured photo-center of an astrometric binary consists of (a) the linear motion of the center-of-mass of the binary, and (b) the wavy orbital motion of the photo-center with respect to the center-of-mass. It is the latter component (b) of the motion which is responsible for the statistical nature of high-precision astrometry. For example, the 'instantaneous' proper motion, measured during a short interval of time, will vary with the epoch of measurement (in accordance with the period of the binary). The typical amplitude of this variation of the proper motion, and similarly the orbital displacement in position, introduce a noticable 'cosmic error' into the measurements of positions and proper motions, as soon as the accuracy of the measurements reach these typical amplitudes of the orbital motions of the photo-centers of astrometric binaries. For example, even if we were able to measure at a certain instant of time the 'instantaneous' values of the position and of the proper motion of the photo-center of an astrometric binary with infinite accuracy, these results would *not* allow us to predict the position of the photo-center at other epochs with high accuracy. It is the 'cosmic noise' in the 'instantaneous' positions and proper motions which limits the accuracy of our predictions.

The 'cosmic noise' mentioned above occurs only if we are unable to determine the complete orbit of the astrometric binary individually. Unfortunately, this will be probably the case for most astrometric binaries, even in high-precision astrometry such as GAIA. The reason is the rather short duration of astrometric missions (e.g. 3 years for HIPPARCOS, or the envisaged 5 years for GAIA) with respect to the orbital periods of most of the binaries.

Let us adopt, for illustrative purposes, the distribution of the orbital periods of G-dwarf binaries, determined by Duquennoy and Mayor (1991), as representative for astrometric binaries in general. Then the fraction of binaries with orbital periods P in the interval 100 days $< P <$ 10 years is about 20 %. Only for this interval of periods, we can hope to determine individual orbits directly from GAIA measurements.

There is another group of astrometric binaries, with periods in the range of 10 years to some 100 years for which GAIA may be able to detect the astrometric-binary nature because of a non-linear motion of the photo-center. Such a procedure has been carried out already successfully for HIPPARCOS, where we determine for some stars significant 'acceleration terms' $g = d\mu/dt$ or even dg/dt

(μ denotes one component of the proper motion, t the time). Unfortunately, these acceleration terms are of limited use for long-term predictions. In most cases, we have to neglect them in practical applications, thereby going back to the 'single-star model' of linear motion. It can be shown by methods similar to those discussed in Section 2 that over long periods of time the linear model produces more accurate results than the model with acceleration terms.

The overall effect of astrometric binaries on all the observed (and apparently single) stars depend also on the fraction F of binaries among the stars. For main-sequence stars of spectral type G or earlier, we know that F is at least 65 %. Due to the incomplete detection of binaries, the true value of F may be as high as 80 % or more. Even if we subtract the detectable visual binaries and the astrometric binaries with individual orbits, the majority of all the remaining GAIA stars will probably be astrometric binaries, either undetected ones or those with acceleration terms of limited use.

For GAIA, I would expect the following numbers: 1–10 % of the GAIA stars may be astrometric binaries for which Keplerian orbits can be determined individually (HIPPARCOS: a few tenth of a percent), and 20–50 % of the GAIA stars may show significant acceleration terms (HIPPARCOS: a few percent). But even if the astrometric-binary nature is not detectable individually, the statistical effect on the sample of all apparently single GAIA stars will be very strong. Already the much less accurate HIPPARCOS mission has provided this insight empirically. For example, the comparison of proper motions from HIPPARCOS and from the FK5 clearly shows that the differences between these proper motions are significantly larger than the measuring errors, because of the 'cosmic errors' introduced by the effect of astrometric binaries. The much more accurate GAIA mission will be much stronger influenced by the astrometric-binary problem.

Earlier studies which discussed the effect of astrometric binaries on high-precision astrometry have been presented by Lindegren (1979), Soederhjelm (1985), Tokovinin (1993), Brosche et al. (1995), and Wielen (1995).

2. A COHERENT SCHEME FOR STATISTICAL ASTROMETRY

It is the aim of 'statistical astrometry' to incorporate the statistical effects of astrometric binaries into astrometric procedures, such as the comparison of proper motions given in two independent astrometric catalogues or the prediction of a position at a certain epoch from a position and proper motion given in an astrometric catalogue. We have developed a coherent scheme for such a statistical astrometry. Details will be published elsewhere (Wielen, in preparation). Here, we would like to outline the basic idea only.

The fundamental tools for statistical astrometry are correlation functions between the orbital displacements in position and velocity at two instants of time. We denote by $\Delta x(t)$ the difference between the actual position of the photo-center and its 'mean' value at time t. The 'mean' value is the long-term average of the position of the photo-center. For circular orbits, the 'mean' position coincides with the center-of-mass of the astrometric binary. In most cases of non-zero eccentricity, the 'mean' position is slightly different from the center-of-mass. Furthermore, we denote by $\Delta v(t)$ the difference between the actual velocity of the photo-center at time t and the velocity of the center-of-mass. We now introduce the following (auto-)correlation functions:

$$\begin{aligned} \xi(\Delta t) &= <\Delta x(t)\Delta x(t+\Delta t)> \ , && (1)\\ \eta(\Delta t) &= <\Delta v(t)\Delta v(t+\Delta t)> \ , && (2)\\ \zeta(\Delta t) &= <v(t)\Delta x(t+\Delta t)> \\ &= -<x(t)\Delta v(t+\Delta t)> \ . && (3)\end{aligned}$$

The average is an ensemble average over the sample of stars under consideration, but may be partially also envisaged as an average over the time t. In any case, the correlation functions ξ, η, ζ depend on the epoch difference Δt only, not on the epoch t itself. The special values of ξ and η for $\Delta t = 0$,

$$\begin{aligned} \xi(0) &= <(\Delta x(t))^2> \ = \ ((\Delta x)_{\rm rms})^2 \ , && (4)\\ \eta(0) &= <(\Delta v(t))^2> \ = \ ((\Delta v)_{\rm rms})^2 \ , && (5)\end{aligned}$$

define the quantities $(\Delta x)_{\rm rms}$ and $(\Delta v)_{\rm rms}$, which we call the 'cosmic errors' of x and v. There is no correlation between Δx and Δv for $\Delta t = 0$:

$$\zeta(0) = 0 \ . \qquad (6)$$

For strictly 'instantaneous' measurements of x and v, there are relations between the correlation functions:

$$\begin{aligned} \zeta(\Delta t) &= -d\xi/d(\Delta t) \ , && (7)\\ \eta(\Delta t) &= d\zeta/d(\Delta t) \ = \ -d^2\xi/d(\Delta t)^2 \ . && (8)\end{aligned}$$

In the more realistic case of measurements of x and v which are 'averaged' over a finite interval of time (e.g. over about 3 years for HIPPARCOS, or 5 years for GAIA), the correlation functions have to be corrected for this 'averaging'. The relations (7) and (8) are then not strictly valid. The functions $\xi(\Delta t)$ and $\eta(\Delta t)$ are symmetric in Δt and look like cosinus functions with rapidly decreasing amplitudes for increasing $|\Delta t|$. $\zeta(\Delta t)$ is antisymmetric in Δt and looks like a 'damped' sinus function. All the correlation functions should approach zero for $|\Delta t| \rightarrow \infty$.

We present two simple examples for applying these correlation functions. First, we ask for the expected mean error of a predicted position of a star. We assume that an astrometric mission has measured 'instantaneous' values of the position, $x(t_1)$, and of the velocity, $v(t_1)$, at the epoch t_1. We now predict the position $x_p(t_2)$ at time t_2 by using the linear model:

$$x_p(t_2) = x(t_1) + v(t_1)(t_2 - t_1) \ . \qquad (9)$$

Let us, for simplicity, assume that the measuring errors of $x(t_1)$ and $v(t_1)$ are zero. Then the expected rms difference σ_{x_2} between the true position $x(t_2)$ and the position $x_p(t_2)$ predicted by Eq.(9), is given by:

$$\begin{aligned} \sigma^2_{x_2} &= 2(\xi(0) - \xi(t_2 - t_1)) - 2\zeta(t_2 - t_1)\cdot(t_2 - t_1) \\ &+ \ \eta(0)\cdot(t_2 - t_1)^2 \ . && (10)\end{aligned}$$

σ_{x_2} may be called the 'cosmic error' of $x_p(t_2)$. For $|\,t_2 - t_1| \rightarrow \infty$, the last term with $\eta(0)$ is the largest one. For small values of $|t_2 - t_1|$, Eq.(10) takes correctly into account that σ_{x_2} is extremely small, because then the linear prediction by Eq.(9) is very accurate.

In our second example, we compare two instantaneous velocities (or proper motions), $v(t_1)$ and $v(t_2)$, measured for the same star at two different epochs t_1 and t_2. We neglect again the measuring errors of $v(t_1)$ and $v(t_2)$. In the linear model, $v(t_1)$ and $v(t_2)$ would be equal. In the presence of astrometric binaries in our sample of stars, $v(t_1)$ and $v(t_2)$ would differ in general. The expected rms difference is given by:

$$\begin{aligned} \sigma^2_{v_2-v_1} &= <(v(t_2)-v(t_1))^2> \\ &= 2(\eta(0)-\eta(t_2-t_1)) \ . \end{aligned} \tag{11}$$

In the limit $|t_2 - t_1| \to \infty$, we obtain $\sigma^2_{v_2-v_1} = 2\ \eta(0)$. In the limit $|t_2 - t_1| \to 0$, both velocities do agree. This is correctly described by Eq. (11), since in this case $\eta(t_2 - t_1) \to \eta(0)$ and hence $\sigma_{v_2-v_1} \to 0$. In both examples, we have not translated the position and velocity into an angular position and a proper motion, as it should be done in real applications. Both Eqs.(10) and (11) show that the correlation functions ξ, η, ζ are useful tools for statistical astrometry.

3. ESTIMATES FOR THE COSMIC ERRORS IN POSITIONS AND PROPER MOTIONS

We would now like to demonstrate how important the 'cosmic errors' in positions and proper motions, due to astrometric binaries, are for high-precision astrometric missions. For simplicity, we assume rather rounded absolute values of $(\Delta x)_{\rm rms}$ and $(\Delta v)_{\rm rms}$:

Absolute values assumed:

$$\begin{aligned} &(\Delta x)_{\rm rms} \sim 1\,\text{astronomical unit (AU)}\,, \\ &(\Delta v)_{\rm rms} \sim 1\,\text{km/s}. \end{aligned}$$

These values are educated guesses, based both on the statistics of binaries and on some experience with actual data, and should describe at least the order of magnitude of the effect correctly. $(\Delta x)_{\rm rms} = 1$ AU means that the cosmic error in the angular position is just equal to the parallax of the star. The value of $(\Delta v)_{\rm rms} = 1$ km/s translates into a cosmic error of the proper motion of 0.2 mas/year divided by the distance r of the star in kpc. The cosmic error in position is governed by long-period binaries, while most of the cosmic error in velocity is due to short-period binaries.

For a typical HIPPARCOS star with a distance r = 100 pc (parallax p = 10 mas), we have:

Typical HIPPARCOS star:

$$\begin{aligned} &(\Delta x)_{\rm rms} \sim\ 10\ \text{mas}\,, \\ &(\Delta \mu)_{\rm rms} \sim\ 2\ \text{mas/year}\,. \end{aligned}$$

Hence the cosmic error in position is larger than the measuring error of HIPPARCOS, while the cosmic error in proper motion is of the same order of magnitude as the measuring error of HIPPARCOS.

For a typical GAIA star, we use a distance r = 1 kpc (parallax p = 1 mas) and a measuring accuracy of 10 μas in position and 10 μas/year in proper motion. We then obtain:

Typical GAIA star:

$$\begin{aligned} (\Delta x)_{\rm rms} &\sim 1\ \text{mas} = 1000\ \mu\text{as} \\ &= 100\times\ \text{measuring error}\,, \\ (\Delta \mu)_{\rm rms} &\sim 0.2\ \text{mas/year} = 200\ \mu\text{as/year} \\ &= 20\times\ \text{measuring error}\,. \end{aligned}$$

It is obvious that the 'cosmic errors' are much larger than the measuring errors for a very high-precision astrometry mission like GAIA.

4. STRATEGY FOR HIGH-PRECISION ASTROMETRY MISSIONS

The basic implication of the high 'cosmic errors' of positions, caused by the orbital motions of astrometric binaries among the target stars of high-precision astrometry missions like GAIA, is the following: The modelling of the motion of each star should be as accurate as possible. Instead of a purely linear motion, one should try to fit the observations either with an additional Keplerian orbit (short-period binaries) or with additional acceleration terms (binaries with longer periods). Only if the modelling errors are kept small with respect to the measuring errors, the full capacity of the mission, e.g. with respect to the measurement of accurate parallaxes, can be achieved. In order to be able to model the motion of a star as accurate as necessary, the number of observations per star should be as large as possible: at least 50, better 100 or more observations per star. For many astrometric binaries observed by HIPPARCOS, the number of observations turned out to be too small for a reliable orbit determination.

5. IMPLICATIONS OF THE COSMIC ERRORS FOR APPLICATIONS OF THE RESULTS OF HIGH-PRECISION ASTROMETRY MISSIONS

The 'cosmic errors' of positions and proper motions measured by high-precision astrometry should be properly taken into account, if we consider which astronomical problems can be solved by missions like GAIA. We shall discuss here only a few examples.

(1) *Galactic kinematics:* A random cosmic error of about 1 km/s in the space velocity of stars is harmless for galactic kinematics in general, e.g. for the determination of the galactic rotation curve or for deriving velocity dispersions of groups of stars. Hence galactic kinematics on large scales is not affected by the 'cosmic errors' in proper motions (nor in position, of course).

(2) *Open star clusters:* The internal velocity dispersion of most open star clusters is smaller than 1 km/s. Therefore, the 'cosmic errors' in velocity (i.e. in proper motion) are quite disturbing for studies of the internal kinematics of open clusters. The small velocity dispersions derived for some clusters from ground-based observations (e.g. about 0.1 km/s for the Ursa major cluster by Wielen (1978a,b)) are nevertheless real, because ground-based astrometric catalogues (such as the FK5) average over a very long interval of time, thereby determining effectively the motion of the center-of-mass of the stars.

(3) *Reference frame using stars:* The reference frame derived and used by missions like GAIA will be defined by

quasars, which do not suffer from the 'cosmic errors' discussed in the earlier sections. If we would like to make the reference system more dense by adding stars, then we should be aware of the 'cosmic errors' in positions and proper motions. One way to minimize the effect of the 'cosmic errors' is to choose as reference stars those objects which have the largest distances r from the Sun. Since the 'cosmic errors' are approximately constant in absolute values, because the binary motion itself does not depend on distance, the 'cosmic errors' in angular position and in proper motion should decrease roughly proportional to the parallax of the stars. While this is not strictly true in a magnitude-limited sample, large distances r still imply small 'cosmic errors' on average. Since GAIA itself is providing accurate parallaxes, the choice of suitable reference stars with large distances r should be possible in most areas of the sky.

6. GENERAL CONCLUSIONS

High-precision astrometry missions such as GAIA shall give an enormous amount of scientific information on astrometric binaries. Some of the most interesting objects may be re-observed from ground-based observatories as spectroscopic binaries. Our knowledge on binaries in general and on stellar masses would be very much improved.

Astrometric binaries represent an interesting challenge for the planning of a high-precision astrometric mission, for the data reduction procedure, and for some of the astronomical applications.

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