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Orbits of real and fictitious asteroids studied by numerical integration

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Abstract. The paper starts with a review of the author's various numerical studies on asteroid orbits, ruled by the violent evolution of the computer technique, and continues with a collection of starting values of orbital elements. This collection supplements the author's numerous papers on orbits at resonances of mean motion with respect to Jupiter. Especially, it refers to work on Trojan-type motion, mainly done together with R. Bien, and to the Hilda and Hecuba cases of resonance. It will allow the extension of intervals covered by numerical integration in interesting cases. The collection contains hitherto unpublished examples of orbits and additional comments. In particular, special remarks and some new results refer to low-eccentricity motion of Hecuba type.

Key words: minor planets – celestial mechanics

1. Introduction.

During about 30 years of scientific activity at the Astronomisches Rechen-Institut, I have studied numerous orbits of real and fictitious asteroids by numerical integration of the rigorous equations of motion that result from the Newtonian forces of attraction of the sun and one or several planets. Resonances of mean motion with respect to Jupiter have become a major topic among my interests, and the present paper mainly refers to orbits that are related to this topic. However, in Section 2 I give a more general review of my studies.

I have used the n-body program by Schubart & P. Stumpff (1966) for most of my integrations that depend on rigorous equations of motion. This program offers an easy way of defining an orbit at an epoch by means of starting values of orbital elements or of rectangular coordinates and velocities. Such a definition is useful for comparisons with other studies, or for future extensions of special computations, but I have frequently omitted a precise definition of this kind in my or joint publications on results about cases of resonance. For instance, Nakai & Kinoshita (1985) have used only similar starting values in an extended integration of orbits corresponding to examples of Bien & Schubart (1984). Now Section 3 presents starting values for a selection of the studied orbits that are related to resonances. Some representative examples replace large sets of orbits. Other sets are augmented by unpublished examples. Additional explanations or comments appear with some orbits, especially with those that are first mentioned. I expect that the collection of starting values will stimulate future studies of the given or analogous sets of orbits by more powerful computing equipment, and that such studies will help to fill the gaps in our knowledge on resonant motion, which are left open due to difficulties of applying analytical theories.

2. Review of the author's numerical studies on asteroid orbits.

The rapid evolution of the available computing facilities has considerably changed the possibilities of applying numerical integration to work on asteroids during my career. I remember to have proceeded with a speed of 3 steps of integration per workday at the former branch of Astronomisches Rechen-Institut in Potsdam-Babelsberg, when I used Cowell's method and a mechanical calculator in work on a differential correction of the orbit of the low-eccentricity asteroid (1453) Fennia (Schubart 1955). However, at that time I have noticed the power of the tool of numerical integration as well as the need of careful test computations. Later on, at Sonneberg Observatory, I got assistance in my work on the rediscovered Amor-type object (1627) Ivar = 1929 SH from Cincinnati Observatory: P. Herget used an IBM–650 electronic computer to derive for me the special perturbations by several planets on this object (Herget 1956a, Schubart 1958). He has also verified my earlier hand computation on Fennia (Herget 1956b). Finally, at the Astronomisches Rechen-Institut in Heidelberg I have had access to electronic computers of increasing power, and the collaboration with staff members or visitors proved to be very valuable. Opportunities for temporary working at important institutions in U.S.A. (see Schubart 1964, 1968) have supported my studies, especially by suggestions and encouragement. As visible from the following subsections, I have proceeded from limited attempts to find periodic solutions to more general studies that required an increasing number of integration steps or attracting bodies.

2.1. Some attempts of approaching periodic solutions.

The efforts of earlier astronomers have stimulated my own interests in studying solutions of the three-body problem and of its subproblems. Applying my own ideas and hand computations, I had succeeded in finding simple types of periodic solutions of special problems by numerical integration (Schubart 1956). In the years following that work I have tried to approach other types of periodic solutions, since a more general way of solving the basic problems appeared to be beyond my possibilities. Using methods of C.L. Siegel, I have even applied power–series developments to prove the existence of new periodic solutions of Hill's Lunar problem (Schubart 1963). I was early led by published work and by correspondence or personal discussions with E. Rabe and K. Stumpff to numerical studies about periodic solutions of asteroid problems at resonances. These studies have only consisted of first attempts to approach such solutions, but I mention them for completeness. They refer to the circular restricted problem sun–Jupiter–asteroid, studied in a rotating reference system. For instance, approaches to periodic Trojan orbits of planar (Schubart 1957) or three-dimensional (see Fricke 1964) type were subsequently replaced by extensive work of Rabe (1961) and Zagouras (1985), respectively. Work by Colombo et al.(1968) has superseded my preliminary efforts of the year 1962 to find periodic solutions in the vicinity of the Hecuba case of resonance. K. Stumpff (1965) has reported on these efforts. Other periodic solutions that have resulted later as a by-product (see, e.g., Schubart 1968, Bien 1980), do not refer to a rigorous three-body problem.

2.2. Real and fictitious orbits at mean-motion resonances.

In my studies on types of motion at resonances, I have mainly considered the Trojan, Hilda, and Hecuba cases which are respectively given by an approximate 1/1, 3/2, and 2/1 ratio of the mean motions of an asteroid and Jupiter. In nature there are Trojan and Hilda groups of asteroids, but the Hecuba case corresponds to one of the gaps in the statistics of mean motion. In 1963 in U.S.A., I have started to study the general case of motion at several resonances, following ideas of Poincaré (1902) and personal suggestions by I. Izsak. At that time I have used the circular restricted problem sun–Jupiter asteroid, but with simplifications of the disturbing function by means of an averaging procedure (Schubart 1964, 1966a). In 1965 D. Brouwer proposed to me an exploration of orbits in the vicinity of the 3/2 resonance, with particular attention to the Hilda-type asteroids. In doing so, I have again simplified the basic problem by removing short-period effects: I have applied averaging and then numerical integration to differential equations that correspond to the planar elliptic restricted three-body problem (Schubart 1968). This numerical method was useful in a first exploration of the evolution of eccentric Hilda orbits. In 1976 I have extended this method to three-dimensional motion (Schubart 1978). All these methods of averaging are only applicable to orbits in the vicinity of a specified resonance. They approximate the real motion, unless close approaches to Jupiter occur.

Considering the importance of numerical studies on resonances in connection with asteroids, I have continued with such studies. I am grateful for the opportunities to do so at the Rechen-Institut. To come closer to the conditions of nature, I have gradually come back to the integration of the rigorous equations of motion at cases of resonance (Schubart 1970, 1979, 1982b, Bien & Schubart 1983a). Later on R. Bien and I have applied a method of digital filtering (Bien & Schubart 1983b, Schubart & Bien 1984) on the results of integrations. This way has now completely replaced the earlier procedure of integrating simplified equations. Eventually, the addition of Saturn or further remote planets to the attracting bodies has led to more realistic studies. This refers, for instance, to the derivation of characteristic orbital parameters from results of extended integrations of the orbits of real Trojans (Bien & Schubart 1987) and Hildas (Schubart 1991), but as well to other recent studies on real or fictitious orbits at the 2/1, 3/2, and 4/3 resonances and on some cases of analogy (Schubart 1988, 1990, 1993). The collection of starting values presented in Section 3 refers to the work on resonances that is based on rigorous equations of motion.

2.3. Orbits fitted to observations in special projects.

The n-body program mentioned in Section 1 is especially useful for the simultaneous integration of the preliminary orbit of a small object and of similar orbits that differ by the variation of one or several starting parameters. Special cases of fitting an orbit to an extended set of observations require such orbits of variation. For instance, I had to vary the uncertain orbital period in a former set of elements of a comet to fit its orbit to very remote observations of the past (Schubart 1966b). In that work the sun and the planets from Jupiter to Neptune represented the massive bodies. Accurate work on asteroid orbits needs the addition of further planets. In several cases I have fitted the orbit to observations in a differential correction with seven unknowns. Then six orbits of variation represent the influence of changes in the six orbital elements. A seventh orbit, separately integrated, gives the effect of a change in a considered value of mass. The deviations of these orbits from a preliminary orbit of reference allow the computation of the coefficients in the equations of condition, that are solved by the method of least squares for the seven unknown corrections to the elements and the mass. Zech (1968) has described this method by formulas.

A first application of this procedure turned out to be necessary for a correction of the orbit of the earth-approaching asteroid (1221) Amor, since a theoretical representation of its observations requires both an accurate method and an accurate value of the mass of earth + moon. A correction to Newcomb's value of this mass and a satisfactory representation of the available observations has resulted from my work on Amor (Schubart 1969). Later on I have applied the procedure to determine or correct values of mass of large asteroids. Earlier work by H.G. Hertz and test computations with the n-body program had demonstrated a few possibilities to determine the mass of an asteroid from gravitational effects caused in the motion of a suitable other asteroid (Schubart 1971, 1972). I have first determined or corrected the masses of (1) Ceres, (2) Pallas, and (4) Vesta, considering the orbits and observations of Pallas or Vesta, Ceres, and (197) Arete, respectively. For references on this work see Schubart (1992). The resulting orbits have only had a temporary value, but I have published special sets of starting values in papers about the used observations (Schubart 1976, 1992).

3. Collection of starting values and supplementary remarks.

Almost all the orbits of asteroids collected in this section correspond to simple mean-motion resonances with respect to

Jupiter. The real or fictitious asteroid moves as a massless member of a three- or several-body system. The models of the forces, given by the massive bodies of a system, are described first. Then the orbits of the small objects follow in groups, separated according to the main resonances. The sequence of the former studies determines the order in a group. Osculating orbital elements appear in each case. The designation corresponds to the permanent number in case of a real asteroid. A special system of numbers refers to the fictitious orbits. These numbers are marked by a preceding F. An asterisk following a number indicates that the accurate definition of the orbit requires a comparatively large number of digits. These orbits are collected in a special table (Table 3), but they appear at reduced accuracy in the groups as well. To avoid ambiguity, preliminary orbits of two real objects are marked in the designation by a following p. Table 9 relates orbits and publications on results by code numbers. Beginning with the fictitious orbits, Table 10 shows the length of the intervals of time covered by the forward and backward integrations. The standard accuracy used in most of these integrations corresponds to about 16 decimal digits (double precision).

The sets of starting values considered here consist of six osculating orbital elements: ω = argument of perihelion, Ω = longitude of ascending node, i = inclination, M = mean anomaly at epoch of osculation, a = semi-major axis, e = eccentricity. The original values have a comparatively low accuracy in general, due to rounding of unnecessary digits. Values of higher accuracy have resulted, if elements were earlier transformed to a new plane of reference, or if given initial coordinates and velocities were now changed to orbital elements, to obtain a uniform presentation in this paper. The epoch of osculation and the unit of a correspond to the choice made for the massive bodies in each case. The orbital elements are "heliocentric". Strictly speaking, they refer to the barycenter of the sun, or of the sun and the inner planets, if their masses are combined in the central body. Further general or specific symbols include: $\varpi = \omega + \Omega$ = longitude of perihelion, $\ell = M + \varpi$ = mean longitude, $\sigma = (p+1)\,\ell_J - p\ell - \varpi$, where p is a small integer. The suffix J refers a symbol to Jupiter. σ is the critical argument of a case of resonance, if that corresponds to an approximate $(p+1)/p$ ratio of the mean motions of asteroid and Jupiter.

3.1. The massive bodies.

Two types of models for the forces of attraction have been used. Type J, the earlier one, is useful in work on the sun-Jupiter-asteroid problem, but I have applied it in more general problems as well. Type A makes use of standard units like AU. In 1984 R. Bien and I have changed to type A, for greater convenience in our project on the orbits of real Trojans. In models of type J, the gravitational constant, the solar mass, and a_J are taken equal to unity. The attraction of the inner planets is entirely neglected, so that the mass of the central body equals one. The masses of Jupiter and Saturn are respectively equal to 1/1047.355 and 1/3498.5, and correspond to the IAU(1976) System of Constants. The unit of time is fixed by the choices already made. I have used a value of August 1975, $a_J = 5.202345$ AU, to change distances to AU, and I have multiplied intervals, given in the unit of time, with a factor 1.8885841, to change them to tropical years.

Models of type A correspond to the proposals on units and mass values made by Schubart & Stumpff (1966) in the publication on the n-body program that was used in the integrations. The units of length and time are AU and 40 ephemeris days, respectively. Table V of Schubart & Stumpff (1966) shows the necessary mass parameters. The masses of the sun and the inner planets are combined in the central body. In units of solar mass, the mass of the central body equals 1.000 00 597682, and the reciprocal values of mass of the planets from Jupiter to Neptune are equal to 1047.355, 3501.6, 22869, 19314, respectively. Note that the value for Saturn corresponds to an earlier IAU System of Constants in this case.

Table 1 presents starting values for six models of type J and nine models of type A. A single line, or the first line of a model corresponds to Jupiter. A letter C or E in the designation points to a circular or elliptic restricted three-body problem sun-Jupiter-asteroid. A designation starting with S indicates a model that includes the attraction of Saturn. Its starting values appear in a second line. Models S1, S16–S19, and SC1 consider the inclination of the orbit of Saturn with respect to that of Jupiter. Model N06 is an extension of S06: Uranus and Neptune are added, line 3 and 4 show the respective starting values. Model E was generalized to S0 and S1 with starting values of Saturn that correspond to an epoch in August 1975 with approximately $M_J = 0°$, $e_J = 0.048$. The notes to Table 1 show epochs that refer to some of the models. SC0 and SC1 are fictitious low-eccentricity models of the perturbing planets. They correspond to S06 and S16, since, for instance, the mean longitudes have similar values.

The initial orbital plane of Jupiter is the reference plane for all models of Table 1, and the zero direction of longitude corresponds to the initial direction of ϖ_J in many cases. However, models SIP and NIP presented in Table 2, have resulted from S1 and an extension of S16 by a rotation, so that the reference plane is given by the invariable plane of the considered system of massive bodies. In these cases, the zero direction of longitude differs by 180° from the initial value of Ω_J. Accurate starting values of three Trojan orbits that were rotated in the same way, appear in the upper part of Table 3, see Table 4. Tables 4–6 and 8 refer each asteroid orbit to one of the models, but many orbits were integrated in two or three models. In particular, this has occurred in studies on differences between the results of related models, like S0 and S1, or S06 and S16.

3.2. Trojan-type orbits.

The list of orbits shown in Table 4 starts with orbit F1*, an example of temporary libration about the Lagrangian libration point that precedes Jupiter in its motion. This example was studied by Rabe and Schubart in Heidelberg in August 1962. We have mentioned it without details in several papers, see the first line of Table 9. I have recently repeated the forward integration in high accuracy. After three cycles of libration of large amplitude about the preceding point, the object escapes to the domain of the following libration point, where temporary libration occurs again.

Bien and Schubart have studied the remaining orbits of Table 4 in an effort to understand the most important effects in the long-period evolution of the orbits of real Trojans and of analogous fictitious objects. Orbits F2–F7 are typical examples of a larger set of orbits that was studied first. Other examples correspond to further variations of the initial values of i, e, or M of orbits F3 and F4. The studies of this set refer to model E in most cases, and to both S0 and S1 in case of F2–F5, to obtain information on differential effects. Orbits F9 and F10 demonstrate in rounded values, together with F8*, how the initial value of Ω was varied by multiples of 45°.0 in the original work, while ϖ was retained. Nakai & Kinoshita (1985) have studied orbits similar to two orbits of this work. Orbits F8* — 1871* refer to work on effects by secular resonances. Studies on differential effects in the evolution of i and Ω of orbit F15 by models SC0 and SC1 have demonstrated, that the influence of the eccentricities of Jupiter and Saturn on these effects is comparatively small. The following orbits of real Trojans are a part of those that were investigated by Bien & Schubart (1987) for the derivation of orbital parameters, which characterize an orbit during long periods. For a recent, more refined way of deriving such characteristic parameters see Milani (1993). All these orbits were studied by models S06 and S16 at least. Orbits 624, 659, 1870, 1873, 2146, and 2594 were integrated in model N06 as well.

3.3. Hilda-type orbits.

Orbit 334p in Table 5 gives an example of an object that is, by its mean motion, only moderately close to a resonance. Its eccentricity varies at small values. In the respective study (Schubart 1970) I have early noticed a way of approximately describing the variations of e and ϖ of such objects by simple formulas with suitable constants. See the following subsections for remarks on analogous objects in the neighborhood of the Hecuba resonance. Both orbits F16 and F17 show a simultaneous libration of a critical argument σ about 0° and of ω about 90° in both models E and S0. To allow differential studies, 1746* and 1941* moved forward and backward in time according to models S0 and S1. Fictitious examples have resulted from these orbits by putting $i = 0°$ at the start. Most of the remaining orbits of Table 5 refer to my work of deriving three characteristic parameters for orbits of Hilda type. Orbit F20 and similar examples obtained by the variation of the starting value of a have demonstrated in a simple model the transition from permanent libration of σ to temporary libration or circulation. A study of the second orbit of (334) Chicago by models S06 and S16 has revealed a non-quasiperiodic evolution of an interesting type. Secondary resonances rule the evolution of orbits F21–F24. Special care is necessary in studies on the low-eccentricity orbits 1256 and 4196, see Schubart (1991).

3.4. Hecuba-type orbits.

I have studied orbit 1921* of Table 6 in models E and S0. I have added 1922* as an additional body in one of these studies, but I have only used the results to confirm information obtained by a comparatively simple model. I have integrated orbits F25–F27 in an effort to find simple cases of analogy for a temporary type of evolution of 1921*. F25 and F26 show a simultaneous libration of σ and ϖ, F27 a slow variation of ϖ, which indicates circulation of this argument. In case of F28, σ changes between libration and circulation. I have studied F29 more recently, together with F21 and F22. There is an influence of secondary resonances on F29 as well, but a temporary one. Most of the following orbits of Table 6 are examples of low-eccentricity motion with values of a that are somewhat smaller than the value corresponding to an exact 2/1 resonance ratio. I have first studied 903 as a case of analogy to low-eccentricity motion at other resonances. Starting from a similar orbit, I have then approached the exact resonance by a sequence of orbits. F30 and F31 are members of this sequence. F31 shows a non-quasiperiodic evolution that is temporarily ruled by the influence of a special type of secondary resonance. F32 demonstrates this special type in a simple model. In November 1992 I have integrated orbits 1101 and 5360, see the following subsection for results.

3.5. Supplementary remarks on Hecuba-type orbits of low eccentricity.

In work on low-eccentricity motion near the 2/1 resonance (Schubart 1993), I have used an empirical transformation from e, ϖ, and σ to new variables, to remove an important part of the effects by the eccentricities of Jupiter and Saturn in models like S16 or S05. I have succeeded to do so by a suitable choice, for each orbit, of a constant κ in the transformation. Since now I prefer to call the new variables e_r, ϖ_r, $\overline{\sigma}$ in case of the 2/1 resonance, I repeat the formulas of the transformation:

$$e_r \cos(\varpi_r - \varpi_J) = e \cos(\varpi - \varpi_J) - \kappa \, e_J \; ;$$
$$e_r \sin(\varpi_r - \varpi_J) = e \sin(\varpi - \varpi_J) \; ;$$
$$\overline{\sigma} \; = \; \sigma + \varpi - \varpi_r \; .$$

In earlier work on Hilda-type asteroids (Schubart 1968, 1982b), I have called a variable e_p that corresponds to e_r, proper eccentricity. However, this designation has another meaning in the theory of the motion of non-resonant asteroids, and there are natural cases of transition to non-resonant motion near the 2/1 resonance. In this context I point to my results on low-eccentricity Hilda orbits (see Section 4 of Schubart 1991), which are analogous to cases of transition, since they have small mean values of a, $\overline{a}$, in comparison with the value of exact resonance (Schubart 1990). In the evolution of the elements of these orbits, effects that are characteristic for the proper parameter $\overline{e}_p$, do not correspond to the secular effects of non-resonant asteroids. Now I propose to change the designation of the corresponding characteristic parameter of the 2/1–resonance case from $\overline{e}_p$ to $\overline{e}_r$, to avoid an abbreviation of proper eccentricity. I retain the designation $\mathcal{A}$ (Schubart 1993) of a second parameter needed in the description of the main variations of e_r and ϖ_r according to Section 4 of Schubart (1991), but I drop a former restriction of the amount of $\mathcal{A}$.

Using the procedure of Section 4 of the earlier paper (Schubart 1991), I have recently determined the characteristic parameters $\overline{e}_r$ and $\mathcal{A}$ of orbits 1101 and 5360 of Table 6. As an addition to Table 1 of Schubart (1993), I present the results on these and other parameters of the two orbits in Table 7. For comparison, I repeat values on orbit 903 by Schubart (1993). Note the similarity of the values of corresponding characteristic parameters of (1101) Clematis and (5360), a recently numbered asteroid. Contrary to the results on (903) Nealley, $\mathcal{A}$ dominates in amount with respect to $\overline{e}_r$ in the two other cases. I have earlier found an analogous result for the evolution of orbit 334p of Table 5. Maybe there are more cases of similarity of the characteristic orbital parameters among the asteroids near the 2/1 resonance.

3.6. Some orbits of special types.

Three of the orbits presented in Table 8 correspond to a 4/3 ratio of resonance. Orbits F33 and F34 are cases of analogy to F16 and F17. They show a simultaneous libration of the respective argument σ about 0° and of ω about 90°, again in both models E and S0. Natural objects that correspond in type to these orbits have not been found. (279) Thule is the natural prototype of this resonance. I have studied its orbit by both models S06 and S16.

Orbits F35 and F36 correspond to a 7/5 ratio of resonance and refer to a simple model. I have studied them to find at this resonance examples of libration of a modified critical argument τ that depends on Ω instead of ϖ, see Schubart (1964, 1978). For the 7/5 resonance, 2τ equals $7\ell_J{-}5\ell{-}2\Omega$. In both cases a short forward integration shows libration of 2τ about 180°, but this libration can be temporary, especially in case of F36.

Looking for cases of analogy to the special evolution of the second orbit of (334) in Table 5, I have studied the two last orbits of Table 8. Indeed, there is some analogy in the evolution of the orbit of (522) Helga.

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