LOW-ECCENTRICITY MOTION OF ASTEROIDS NEAR THE 2/1 JOVIAN RESONANCE

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Abstract

(903) Nealley moves on an orbit of low eccentricity with a mean motion that is slightly larger than the 2/1 value of resonance. This orbit and some related fictitious orbits are studied by numerical integrations of the four-body problem Sun-Jupiter-Saturn-asteroid over an interval of 110 000 yr. The author's experience on related cases of resonance allows a study of the variation of suitably defined orbital parameters. The long-term evolution of the orbits is compared with earlier predictions. Some of the librating orbits are temporarily captured in a secondary resonance that refers to three-dimensional motion and is demonstrated by a special example.

Keywords: Asteroids, evolution of orbits, 2/1 resonance, proper parameter, secondary resonance.

1 Introduction

The evolution of asteroidal orbits in the vicinity of the 2/1 Jovian resonance of mean motion is a subject of continuing interest, see the references given in the respective section of a related paper by Yoshikawa (1991). Many studies on this subject depend on the elliptic restricted three-body problem Sun-Jupiter-asteroid treated in two or three dimensions, but Yoshikawa (1991) points to the importance of including the action of Saturn in the model of the forces, especially in work on the 2/1 Kirkwood gap. It is comparatively easy to consider both Jupiter and Saturn in studies that use numerical integration to derive the motion of massive and small bodies in a simultaneous computation. I have done this in work on resonant motion in the outer asteroid belt, especially on the 3/2 case (Schubart 1988, 1991). During this work I have noticed that the special numerical methods of studying single orbits are applicable to some low-eccentricity orbits near the 2/1 resonance as well, in particular to the orbit of (903) Nealley (Schubart 1988). Now I apply the former methods and experience in a detailed study of this orbit that is situated at the sunward border of the Kirkwood gap in the frequency distribution of semi-major axis. This orbit shows a quasi-periodic behaviour in the interval considered. The same turns out for some related fictitious orbits that are closer to the center of the gap, but seem to have no natural counterparts (compare with Wisdom 1987). In a continuation toward the center of the gap, four further orbits clearly show a non-quasiperiodic behaviour. Frequent changes in the ratio of characteristic long periods appear to be a typical feature of these orbits, that show libration of variable amplitude and temporary capture by a secondary resonance.

2 Numerical Procedures

In the present study I closely follow the definitions and procedures introduced in an earlier paper (Schubart 1988). As before, the integrations of the problem Sun-Jupiter-Saturn-asteroid have resulted from the N-body program by Schubart and Stumpff (1966). All integrations cover an interval of 110 000 yr centered close to the present. I have extended the former integration for (903) Nealley (Schubart 1988) by adding a backward computation in the model '$i_S \neq 0$' that considers the mutual inclination of the orbits of the massive planets. However, the recent integrations on fictitious 2/1 orbits depend on the model '$i_S = 0$' which neglects this mutual inclination. Then Jupiter and Saturn move in the plane of reference that is defined to agree with the real orbital plane of Jupiter at a special epoch. In this model, the lack of long periods that are otherwise caused by the moving planes of the major bodies, allows special applications (see section 3).

As before, I use the symbols $a,\ e,\ i,\ \ell$ for semi-major axis [AU], eccentricity, inclination and mean longitude. ϖ and Ω are the longitudes of perihelion and node. The subscript J refers to an element to Jupiter. The critical argument of the 2/1 resonance is given by $\sigma = 2\,\ell_J - \ell - \varpi$. Again I can use an empirical transformation

$$e_p \cos\,(\varpi_p - \varpi_J) = e \cos\,(\varpi - \varpi_J) - \kappa\ e_J;$$
$$e_p \sin\,(\varpi_p - \varpi_J) = e \sin\,(\varpi - \varpi_J);$$
$$\overline{\sigma} = \sigma + \varpi - \varpi_p$$

to new variables e_p, ϖ_p, $\overline{\sigma}$ with a constant κ that is suitably chosen for each orbit. κ can roughly be fitted in such a way that it approximately removes the asymmetric location with respect to the origin of a corresponding curve plotted in $e \cos\,(\varpi - \varpi_J)$, $e \sin\,(\varpi - \varpi_J)$ coordinates, in the analogous new coordinates. A final adjustment can simplify the resulting variations of $\overline{\sigma}$, see section 3. κ remains comparatively small in the present applications. In this way I try to remove a part of the influence of the eccentricities of the major bodies from e, ϖ, and σ, so that results on the new variables allow a better comparison with analogous results obtained by the circular restricted problem. For example I have found evidence of a permanent libration of $\overline{\sigma}$ for Nealley in this way (Schubart 1988). Proper parameters are given by $\overline{e}_p$, a mean value of e_p, i_p, here a mean value of i, and $\overline{\sigma}_A$, the mean amplitude of the oscillation caused in $\overline{\sigma}$ by the typical period of libration, T_L. I have earlier used these parameters to characterize resonant orbits that show libration of $\overline{\sigma}$ and a quasiperiodic behaviour (Schubart 1991). For low-eccentricity orbits, an additional parameter $\mathcal{A}$ is useful to describe the main variations of e_p and ϖ_p (Schubart 1991, section 4). For instance, e_p approximately equals the length of a two-dimensional vector given by the sum of a constant vector of length $\overline{e}_p$ and of a shorter vector of length $\mathcal{A}$ with uniform rotation according to T_L. Let $\overline{a}$ be a mean value of a that refers to the interval covered by integration in general. I apply digital filtering in the derivation of all these parameters, see Schubart and Bien (1984) and Schubart (1991).

3 Orbital Evolution of Nealley and some Related Examples of Motion

The way of starting the computation for (903) Nealley resembles my earlier procedures (Schubart 1990, 1991). The starting epoch is JDE 2446000.5. The starting elements of Jupiter and Saturn correspond to Table 1 of Schubart (1990), but the orbit of Saturn is ro-

Table 1: Starting values, parameters, and periods

	N = (903)	A	B	C	D	E	
a_0	3.2317	3.240	3.243	3.249	3.255	3.261	[AU]
e_0	0.0701	0.042	0.043	0.046	0.050	0.056	
κ	0.26	0.18	0.163	0.125	0.08	0.04	
$\overline{e}_p$	0.034	0.034	0.036	0.042	0.049	0.057	
$\mathcal{A}$	0.026	0.005	0.005	0.006	0.006	0.005	
i_p	11.°20	11.14	11.14	11.14	11.14	11.°13	
$\overline{\sigma}_A$	54°	10	10	10	10	9°	
$\overline{a}$	3.2387	3.2395	3.2420	3.2468	3.2514	3.2556	[AU]
T_L	308.4	313.2	330.4	366.9	403.3	435.2	[yr]
T_P	342.4	349.2	375.3	438.6	518.9	625.8	[yr]
T_N	18.0	18.3	18.5	19.2	20.0	21.4	[10^3yr]

Notes to Table 1. The starting values a_0 and e_0 of (903) Nealley and of 5 fictitious orbits refer to a common epoch in the comparatively near future. For the numerical constant κ and the following parameters see section 2. T_L is the period of libration. A mean value is given. The mean periods of the retrograde revolution of the arguments $\varpi_p - \varpi_J$ and Ω are designated by T_P and T_N, respectively.

tated to approximate the real inclination. The forward integration brings σ of Nealley close to zero at an epoch that is later by 117.64 yr. I use this second epoch and the corresponding result for Nealley to start the integration of the fictitious orbits with similar values of the angular variables that are close to $\varpi - \varpi_J = 337°$, $\Omega - \varpi_J = 148°, i = 11.°1$, $\sigma = 0°$. I turn the orbital planes of the major planets into the plane of reference at the second epoch for this integration. Then, I vary the starting value of a, a_0, away from the orbit of Nealley, N, to obtain a sequence of orbits A, B, $\cdots$, E that approaches the center of the Kirkwood gap. In each case I adjust the starting value of e, e_0, to obtain a comparatively small effect of libration of $\overline{\sigma}$ with $\overline{\sigma}_A$ near 10°. Table 1 lists a_0 and e_0, referred to the second epoch, the derived proper parameters and other values of interest, especially mean periods. Note that the periods T_P and T_N correspond to the mean retrograde revolution of the arguments $\varpi_p - \varpi_J$ and Ω. Fig.1 demonstrates the relation between $\overline{e}_p$ and $\overline{a}$. a oscillates about $\overline{a}$ in case of orbits N and A, B, $\cdots$, E, and all the other considered variations indicate a quasi-periodic evolution of these orbits, although orbit E shows comparatively large shifts of a period related to T_L. This behaviour suddenly changes, if I try to continue the sequence of orbits beyond E with values of a_0 from 3.263 to 3.267 AU. Four examples started with such values of a_0 show irregular variations of a and other effects that clearly indicate a non-quasiperiodic evolution, see section 4.

In the derivation of the proper parameters shown in Table 1 I have applied the methods for the main case of Hilda motion to $\overline{\sigma}_A$ and i_p, but a special method for small e_p to $\overline{e}_p$ and $\mathcal{A}$ (Schubart 1991). The frequencies resulting in this method allow the derivation of T_L and T_P. In an attempt to determine $\overline{\sigma}_A$ by digital filtering, a band of frequencies passes the filter together with the one that corresponds to T_L. Therefore, a changing amplitude of $\overline{\sigma}$ appears in a plot versus time, but a final adjustment of κ leads to a sufficiently small variation in this amplitude. Along the sequence of orbits A to E, κ shows a tendency to approach zero, but

Figure 1: Mean values of semi-major axis [AU] versus $\overline{e}_p$, a proper parameter related to eccentricity. $a = 3.276$ AU corresponds to the 2/1 ratio of the mean motions of asteroid and Jupiter. N refers to the orbit of (903) Nealley. The letters from B to E represent fictitious orbits that show small effects of libration. A fifth orbit of this kind, A, closely corresponds to the position of the letter N in the figure.

this can be a particular feature for values of i_p near 11° and small $\overline{\sigma}_A$. Among other effects, I remove by digital filtering from $\overline{\sigma}$ an oscillation that apparently follows a frequency given by the difference of the absolute values of the frequencies that correspond to T_L and T_P. I did not notice such an effect during my former work on Hildas. In case of Nealley, the amplitude of this effect changes from about 6° to 3°, following the cycle of the long-period variation of e_J. I assume that this amplitude is roughly proportional to both e_J and $\mathcal{A}$, according to the smaller variations of this kind shown by orbits A, B, and C.

In applying my method of derivation of i_p, I remove a periodic term that follows the mean period of revolution of the argument $2\ \Omega - 2\varpi_J$, from the filtered results on i, if this is necessary due to the length of this period. In doing so for orbits A to E, I noticed an additional effect depending on the mean period of the argument $2\ \Omega - \varpi_J - \varpi_S$ with an amplitude of less than $0\overset{\circ}{.}05$. Here the suffix S refers ϖ_S to Saturn. The use of the model '$i_S = 0$' for these orbits and the corresponding lack of other very long periods has allowed these small effects to show up in my graphs. Effects of this kind are more important for the variations of i of Trojan asteroids, see Fig. 3a of Schubart and Bien (1986).

4 Non-Quasiperiodic Types of Motion and Comparison with other Work

In Fig.1 the sequence from B to E corresponds to orbits with small effects of libration. Evidently, the sequence A, B, $\cdots$, E develops in analogy to the pericentric branch of periodic orbits of the circular restricted problem, see Fig.3a of Morbidelli and Giorgilli (1990). In this context it is interesting to follow the evolution of the four examples with non-quasiperiodic motion started close to orbit E, see section 3. I study them with $\kappa = 0$, according to the small value found for orbit E. This means the use of the original elements e, ϖ, and of σ. The study shows for the four examples a continuing status of libration of σ with $\mid \sigma \mid < 100°$, but with strong fluctuations of the amplitude, in the considered interval. The strong irregular variations of a and e are correlated. During limited periods the mean values of a and e of an orbit roughly correspond to members of the pericentric branch mentioned above.

The importance of secondary resonances for the evolution of 2/1 resonant orbits was pointed out by Lemaître and Henrard (1990), see Fig. 3b of Morbidelli and Giorgilli (1990). According to Table 1, the ratio $\mathrm{T}_P/\mathrm{T}_L$ develops from values near 1.1 to 1.44, the value of orbit E, and passes rational values of interest. Fortunately, the orbit started next to E with $a_0 = 3.263$, $e_0 = 0.0585$ shows comparatively smooth variations during a large part of the backward computation, which includes the starting epoch. $\mathrm{T}_P/\mathrm{T}_L$ equals about 1.52 in this interval. Maybe the proximity to 3/2 of the ratio $\mathrm{T}_P/\mathrm{T}_L$ gives rise to the wild evolution of this orbit. The same can turn out in much more extended computations on orbit E. It is interesting to note that the onset of non-quasiperiodic types of motion found here, qualitatively agrees to the predictions on the structure of the 2/1 resonance by Murray (1986), who has used a simplified model of the three-body problem and a mapping technique in his work. I think that his simplified model approximates the conditions of low-eccentricity motion, but his model is a planar one. Wisdom (1987, p.268) has demonstrated by the numerical integration of an orbit with $e_0 = 0.05$, that the transition from a three-body problem to a model with four major planets can change predictions on the structure of the 2/1 resonance.

I have studied the evolution of a, e, and i of the four examples with a_0 beyond 3.261 AU by means of smoothed curves, eliminating the influence of all periods that are less than about 6000 yr, see Fig.2 for e. The following results refer to temporary mean values taken from these curves. An increase and subsequent strong changes of a are apparently triggered by the increase of e_J to a maximum in the forward direction of time. e follows with increase and correlated variations in each case. The librating orbits travel about in an a, e domain that contains, for the ratio $\mathrm{T}_P/\mathrm{T}_L$, the typical 2/1 and 3/1 secondary resonances. The smoothed values of a and e reach 3.270 AU and 0.127 with $\mathrm{T}_P/\mathrm{T}_L$ temporarily near 4/1 in one of the cases. I note that the respective backward computation even shows proximity to the 5/1 ratio of $\mathrm{T}_P/\mathrm{T}_L$ at its end. I have observed original values of e of up to 0.19.

In the more remote future e_J goes down to reach a minimum. At about this time the smoothed values of a and e of all the four examples show a tendency to approach a domain with a near 3.262 and e close to or a little less than 0.08. In Fig. 1 this corresponds to the upper part of the right border. However, the simultaneous smoothed results on i approach different values in the interval from $10\overset{\circ}{.}9$ to $13\overset{\circ}{.}1$. This tendency typically appears in case of an orbit started near E with $a_0 = 3.264$, $e_0 = 0.0598$. Since I suspected the influence of a special type of secondary resonance given by the 2/1 ratio of the mean period of revolution of $\omega = \varpi - \Omega$ and T_L, I have plotted $e \sin 2\omega$ versus $e \cos 2\omega$ for the respective interval, without

Figure 2: Four curves smoothed by digital filtering and plotted against time, demonstrate the non-quasiperiodic evolution of the eccentricity of orbits started in an attempt to continue the sequence from A to E. The bars at the upper and lower border indicate maxima and minima of e_J, respectively. Note the approach of the curves to the level $e = 0.08$ near the lower right bar. The respective starting values a_0 [AU] and e_0 of the four orbits are

3.263	0.0585	: heavy line,	3.264	0.0598	: thin line,
3.265	0.0611	: dashed line,	3.267	0.0640	: dotted line.

smoothing. A polar plot for e and 2ω results in this way. Neglecting short-period variations, I find the subsequent directions from the origin to the maxima of e to librate with a large amplitude about the zero direction of 2ω temporarily. I get the same temporary libration of the directions for all of the four examples near the considered moment of time, and for a related orbit integrated in a simpler model (see below and Fig. 3). This libration corresponds to the evolution of a fictitious Hilda-type orbit studied earlier (see Fig. 2 of Schubart 1990). Here the typical example with $a_0 = 3.264$ shows a period of libration of the direction to the maxima of e of about 3500 yr. ω decreases with a temporary period that varies about 1000 yr under the influence of the longer period, which simultaneously causes considerable changes in the mean of e of a cycle of σ libration. This mean varies between 0.100 and 0.055 according to the period of 3500 yr, and the largest maxima of e reach 0.14. Apparently the influence of the considered type of secondary resonance becomes strong during periods of small e_J. Perhaps the periodic repetition of such periods can prevent the escape of e to very large values during longer intervals, in case of my four examples with a non-quasiperiodic type of motion, if basic libration according to a period T_L continues to occur. However, forbidden regions and further restrictions used by Froeschlé and Scholl (1976) for an ergodic orbit of a simplified model, are not available for the present results, due to the more complicated model of the forces. Wisdom (1987, p.269) has mentioned results of an extended integration with perturbations of the four major planets for some 2/1 orbits of special interest. One of these orbits reaches a maximum eccentricity of 0.53 and demonstrates the possibility of large long-period changes in inclination. As visible from his Fig.19, this interesting orbit starts with $e_0 = 0.1$ and a comparatively small inclination. According to this and to the initial part of its evolution, that orbit is not closely related to the fictitious orbits considered in the present paper.

To demonstrate the special type of secondary resonance considered above by a comparatively simple example, I use starting values from a forward integration over an interval of 36505 yr of the orbit with the original starting values $a_0 = 3.264$, $e_0 = 0.0598$. I continue the integration with the following simplifications and changes: The mass of Saturn and the eccentricity of Jupiter are neglected, the zero direction of longitude is changed. I start the continuation with $a_J = 5.20282$ AU, $\ell_J = 0°$, and with $\varpi = 305\overset{\circ}{.}9$, $\Omega = 162\overset{\circ}{.}6$, $i = 13\overset{\circ}{.}3$, $a = 3.254$, $e = 0.092$, $\ell = 15\overset{\circ}{.}4$. σ librates in the continuation with an amplitude that changes between about 40° and 78°. Fig. 3 refers to a part of this continuation and indicates a 1/1 ratio of the mean period of revolution of 2ω and of the period of libration that causes the sequence of maxima of e. A libration of the direction from the origin to subsequent maxima of e about the zero direction of 2ω is visible. During a cycle of this libration of direction large mean values of e occur together with a large amplitude of libration of σ, but the inclination is comparatively small at the respective phase of this cycle.

References

[1] Froeschlé, C. and Scholl, H.: 1976, 'On the Dynamical Topology of the Kirkwood Gaps', Astron. Astrophys. **48**, pp. 389–393

[2] Lemaître, A. and Henrard, J.: 1990, 'On the Origin of Chaotic Behaviour in the 2/1 Kirkwood Gap', Icarus **83**, pp. 391–409

Figure 3: Polar plot for e and 2ω of a librating orbit of the three-dimensional circular restricted three-body problem Sun-Jupiter-asteroid. The curve starts with a heavy line, continues with dashes and ends as a dotted line. An arrow indicates the sense of motion. The plotted curve corresponds to an interval of about 3360 yr. The period of libration causes extremes of e that change in amount. Note the libration of the direction from the origin to subsequent maxima of e about the zero direction of 2ω (see the text).

[3] Morbidelli, A. and Giorgilli, A.: 1990, 'On the Dynamics in the Asteroids Belt. Part II: Detailed Study of the Main Resonances', Celes. Mech. Dyn. Astron. **47**, pp. 173–204

[4] Murray, C.D.: 1986, 'Structure of the 2:1 and 3:2 Jovian Resonances', Icarus **65**, pp. 70–82

[5] Schubart, J.: 1988, 'Resonant Asteroids between the Main Belt and Jupiter's Orbit', Celes. Mech. **43**, pp. 309–317

[6] Schubart, J.: 1990, 'The Low-Eccentricity Gap at the Hilda Group of Asteroids', *Asteroids, Comets, Meteors III*, eds.: C.-I. Lagerkvist, H. Rickman, B.A. Lindblad, M. Lindgren. Uppsala University, pp. 171–174

[7] Schubart, J.: 1991, 'Additional Results on Orbits of Hilda-Type Asteroids', Astron. Astrophys. **241**, pp. 297–302

[8] Schubart, J. and Bien, R.: 1984, 'An Application of Labrouste's Method to Quasi-Periodic Asteroidal Motion', Celes. Mech. **34**, pp. 443–452

[9] Schubart, J. and Bien, R.: 1986, 'On the Computation of Characteristic Orbital Elements for the Trojan Group of Asteroids', *Asteroids, Comets, Meteors II*, eds.: C.-I. Lagerkvist, B.A. Lindblad, H. Lundstedt, H. Rickman. Uppsala University, pp. 153–156

[10] Schubart, J. and Stumpff, P.: 1966, 'On an N-Body Program of High Accuracy for the Computation of Ephemerides of Minor Planets and Comets', Veröffentl. Astron. Rechen-Inst. Heidelberg No. **18**, Verlag G. Braun, Karlsruhe, pp. 1–31

[11] Wisdom, J.: 1987, 'Urey Prize Lecture: Chaotic Dynamics in the Solar System', Icarus **72**, pp. 241–275

[12] Yoshikawa, M.: 1991, 'Motions of Asteroids at the Kirkwood Gaps II. On the 5:2, 7:3, and 2:1 Resonances with Jupiter', Icarus **92**, pp. 94–117