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Indications on the binary nature of individual stars derived from a comparison of their HIPPARCOS proper motions with ground-based data

I. Basic principles

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Abstract. We present a method which provides some information on the possible binary nature on an appararently single star. The method compares the instantaneously measured HIPPARCOS proper motion with the long-term averaged, ground-based proper motion or with the proper motion derived from old ground-based positions and the HIPPARCOS position. Good sources for such ground based data are the FK5 and the GC.

If the proper-motion difference $\Delta\mu$ is statistically significant with respect to its measuring error, the object is very probably a double star. We call then the object a 'delta-mu binary'. If the proper-motion difference is insignificant and if no other information on a binary nature of the object is available, we call such a star a 'single-star candidate'.

We propose a quantitative test for the significance of the observed proper-motion difference. The sensitivity of our method is high: For nearby stars at a distance of 10 pc, the measuring accuracy of the proper-motion difference, expressed as a velocity, is of the order of 50 m/s (basic FK5 stars) or 80 m/s (GC stars). At 100 pc, the mean error of the two-dimensional difference is still 0.5 km/s or 0.8 km/s.

For the FK5 stars, we provide indications on the probable period of the $\Delta\mu$ binaries. If we adopt an orbital period and a mass-luminosity relation, we can use the observed velocity difference to estimate the separation and the magnitude difference between the two components of the binary.

The present paper concentrates mainly on the basic principles of the method, but it provides also a few examples of delta-mu binaries and of single-star candidates for illustration: γ UMa, ε Eri, ι Vir, 47 UMa, δ Pav.

Key words: proper motions – HIPPARCOS Catalogue – double-star nature of objects – delta-mu binaries

1. Introduction

For many purposes it is important to know whether an object is a single star or a binary. There are many conventional methods to detect the binary nature of an object: direct imaging methods (from visual inspection to speckle observations), photometric methods (eclipsing binaries), and detection of orbital motions (spectroscopic and astrometric binaries). The high measuring accuracy of the ESA Astrometry Satellite has added some new methods for detecting binaries, which lead to the component (C) solutions, acceleration (G) solutions, variability-induced movers (V solutions), and stochastic (X) solutions, in addition to the classical orbital (O) solutions (ESA, 1997).

We discuss here an additional method to detect the binary nature of an apparently single star: The new method is based on the comparison of the quasi-instantaneously measured HIPPARCOS proper motion with a time-averaged, long-term proper motion derived either from ground-based observations alone or from a combination of old ground-based positions with the HIPPARCOS position. The basic idea is illustrated in Fig. 1. For a truly single star, the proper motion measured within a short interval of time should agree, within the measuring accuracy, with the proper motion derived from a very long interval. This is in general not true for a binary: Due to the wavy orbital motion of the photo-center of an unresolved astrometric binary, the instantaneously measured proper motion of the object can differ significantly from a long-term proper motion (ideally the motion of its center-of-mass). We have called the difference between the two proper motions the 'cosmic error' of the instantaneous proper motion (Wielen 1995a, b, 1997).

If we detect for an object an individual cosmic error which is significantly larger than the measuring error, then this object is very probably a binary. We call these objects 'delta-mu binaries' ($\Delta\mu$ binaries) because of the proper-motion difference $\Delta\mu$ which has led to the detection of their double-star nature. In the opposite case, i.e. if the individual cosmic error is well within the expectation provided by the measuring errors, then the object is either actually a single star or the orbital motion of the photo-center was too small to be detected with the given measuring accuracy. We call such an object a 'single-star candidate', if there is nothing known to us that actually indicates a binary nature of this object (either from ground-based obser-

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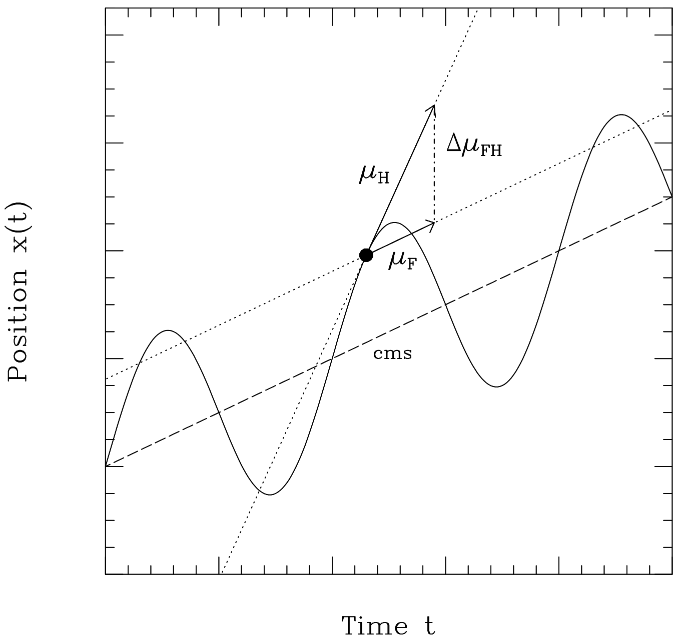

Fig. 1. The wavy motion of an astrometric binary leads to an observable difference $\Delta\mu_{FH}$ between the instantaneously measured HIPPARCOS proper motion μ_H and the mean proper motion μ_F of the star. We assume in this example that the orbital period of the binary is of medium length (e.g. 30 years), so that the proper motion μ_F, obtained from ground-based data (e.g. from the FK5), is essentially equal to the proper motion of the center-of-mass (cms) of the binary.

vations or from HIPPARCOS data). There remains, of course, the third possibility: The cosmic error is neither large enough for qualifying the star as a $\Delta\mu$ binary nor small enough for a single-star candidate.

2. Proper-motion differences

Our method is based on the comparison of an 'instantaneously' measured proper motion with at least one long-term proper motion. Short-term proper motions, μ_H, derived from observations over a period of about 3 years around 1991, are provided by HIPPARCOS (ESA 1997).

There are various possibilities for getting long-term proper motions. Let us start with the FK5 (Fricke et al. 1988, 1991). This catalogue of fundamental stars provides rather accurate long-term proper motions, μ_F, derived from ground-based observations which cover periods of up to more than 200 years. The FK5 gives also mean stellar positions, $x_F(T_F)$. We use the notation x for either the right ascension α or the declination δ of the star. The individual 'central' epoch is chosen such that x_F and μ_F are not correlated. From the HIPPARCOS position, $x_H(T_H)$, with $T_H \sim 1991.25$, and the FK5 position, we can derive a second long-term proper motion μ_0:

$$\mu_0 = \frac{x_H(T_H) - x_F(T_F)}{T_H - T_F} \ . \tag{1}$$

The epoch difference $T_H - T_F$ is typically of the order of 40 years. Since $x_F(T_F)$ and μ_F are uncorrelated, this is also true for μ_0 and μ_F. The same is true for the pair μ_0 and μ_H, if we use the individual central epochs T_H for each star and for the two directions α and δ.

The values μ_F and x_F should not be taken directly from the FK5, since the catalogue values are affected by systematic errors. Instead we have first to reduce the positions and proper motions given in the FK5 to the HIPPARCOS system (see e.g. Wielen 1997, Wielen et al. 1998a, b, based on the method of Bien et al. (1978)). In the case of the FK5 we can now form the differences for the three pairs of the three proper motions, separately for each coordinate component $\alpha \cos\delta = \alpha_*$ and δ. For α_*, we get

$$\Delta\mu_{FH,\alpha*} = \mu_{F,\alpha*} - \mu_{H,\alpha*} \ , \tag{2}$$

$$\Delta\mu_{0H,\alpha*} = \mu_{0,\alpha*} - \mu_{H,\alpha*} \ , \tag{3}$$

$$\Delta\mu_{0F,\alpha*} = \mu_{0,\alpha*} - \mu_{F,\alpha*} \ , \tag{4}$$

and three similar equations for δ. The differences in the two directions α and δ can be added together to a 'total' difference:

$$\Delta\mu_{FH,tot} = \left((\Delta\mu_{FH,\alpha*})^2 + (\Delta\mu_{FH,\delta})^2 \right)^{1/2} \ , \tag{5}$$

and similar equations for the pairs $0H$ and $0F$. Because the central epochs T are usually different for α and δ, the difference $\Delta\mu_{tot}$ is not strictly a proper-motion difference for a certain epoch. This complicates slightly the physical interpretation of $\Delta\mu_{tot}$. But any choice of a common epoch for α and δ would introduce correlations which would disturb our statistical considerations in Section 4.

Already for the FK5, the proper motion μ_0 is usually more accurately measured than μ_F. This is due to the rather good accuracy of old ground-based observations coupled with their high epoch difference with respect to HIPPARCOS. An important compilation catalogue of such older observations is the GC (Boss et al. 1937). The GC contains many more stars (33 342) than the FK5 (4 652), albeit with lower accuracy. Unfortunately the proper motions, μ_{GC}, given in the GC have such large measuring errors that they can not be used for our purpose in most cases. However, the proper motion $\mu_{0(GC)}$, based on the GC position $x_{GC}(T_{GC})$ at $T_{GC} \sim 1900$,

$$\mu_{0(GC)} = \frac{x_H(T_H) - x_{GC}(T_{GC})}{T_H - T_{GC}} \ , \tag{6}$$

is usually accurate enough for being used in our method. Since we have now only two proper motions, $\mu_{0(GC)}$ and μ_H, for a comparison, our formerly three pairs of proper-motion differences are reduced to one difference in the case of the GC:

$$\Delta\mu_{0(GC)H,\alpha*} = \mu_{0(GC),\alpha*} - \mu_{H,\alpha*} \ , \tag{7}$$

$$\Delta\mu_{0(GC)H,\delta} = \mu_{0(GC),\delta} - \mu_{H,\delta} \ , \tag{8}$$

and

$$\Delta\mu_{0(GC)H,tot} = \left((\Delta\mu_{0(GC)H,\alpha*})^2 + (\Delta\mu_{0(GC)H,\delta})^2 \right)^{1/2} . \tag{9}$$

Most of the FK5 stars are also contained in the GC. For these common stars, a comparison of $\Delta\mu_{0H}$ and $\Delta\mu_{0(GC)H}$ provides a partial consistency check. One should remember, however, that the FK5 and the GC often use the same old observations and differ only in the detailed treatment of these common data.

Up to now, we have implicitely assumed that the coordinates α_* and δ of a star change linearly in time. In real applications, the effects of sphericity and the foreshortening effect have to be taken into account. This must be done, however, already for the determination of the systematic differences between the FK5 (or the GC) and HIPPARCOS. If we use later always the HIPPARCOS results as reference values, i.e. $x_F - x_H$ instead of x_F and $\mu_F - \mu_H$ instead of μ_F, then the non-linear effects are already accounted for to a high degree of approximation.

3. Statistical significance

After having derived in Section 2 the proper-motion difference for a given star, we have now to decide whether or not this difference is statistically significant with respect to its measuring error.

The measuring error $\varepsilon_{\Delta\mu,FH,\alpha*}$ of $\Delta\mu_{FH,\alpha*}$ is given by

$$\varepsilon^2_{\Delta\mu,FH,\alpha*} = \varepsilon^2_{\mu,F,\alpha*} + \varepsilon^2_{\mu,H,\alpha*} \ . \tag{10}$$

$\varepsilon_{\mu,H,\alpha*}$ is the measuring error of $\mu_{H,\alpha*}$, given in the HIPPARCOS Catalogue. The measuring error $\varepsilon_{\mu,F,\alpha*}$ of $\mu_{F,\alpha*}$ consists of two parts:

$$\varepsilon^2_{\mu,F,\alpha*} = \varepsilon^2_{\mu,F,\alpha*,ind} + \varepsilon^2_{\mu,F,\alpha*,sys} \ . \tag{11}$$

The first part is the random ('individual') mean error of $\mu_{F,\alpha*}$, provided by the FK5. The second part describes the uncertainty of the reduction of the FK5 system of proper motions to the HIPPARCOS system for the star under consideration. We emphasize that this is not the systematic difference of the FK5 itself, but only the mean error of the determination of this systematic difference.

The mean error $\varepsilon_{\mu,0,\alpha*}$ of $\mu_{0,\alpha*}$ is obtained from

$$\varepsilon^2_{\mu,0,\alpha*} = \frac{\varepsilon^2_{x,H,\alpha*} + \varepsilon^2_{x,F,\alpha*,ind} + \varepsilon^2_{x,F,\alpha*,sys}}{(T_{H,\alpha*} - T_{F,\alpha*})^2} \ . \tag{12}$$

$\varepsilon_{x,H,\alpha*}$ is the measuring error of the HIPPARCOS position $x_H(T_H)$ in α_*, $\varepsilon_{x,F,\alpha*,ind}$ is the random error of $x_F(T_F)$, and $\varepsilon_{x,F,\alpha*,sys}$ is the uncertainty in the reduction of the FK5 system of positions to the HIPPARCOS system for this star. The measuring errors of the proper-motion differences for the pairs $0H$ and $0F$ are then given by

$$\varepsilon^2_{\Delta\mu,0H,\alpha*} = \varepsilon^2_{\mu,0,\alpha*} + \varepsilon^2_{\mu,H,\alpha*} \ , \tag{13}$$

$$\varepsilon^2_{\Delta\mu,0F,\alpha*} = \varepsilon^2_{\mu,0,\alpha*} + \varepsilon^2_{\mu,F,\alpha*} \ . \tag{14}$$

The corresponding equations for the coordinate δ have the same form: α_* is just replaced by δ. Equations (10)-(14) are applicable when using the FK5. Equations (12) and (13) can be easily adapted to the case of the GC by replacing F by GC and 0 by $0(GC)$.

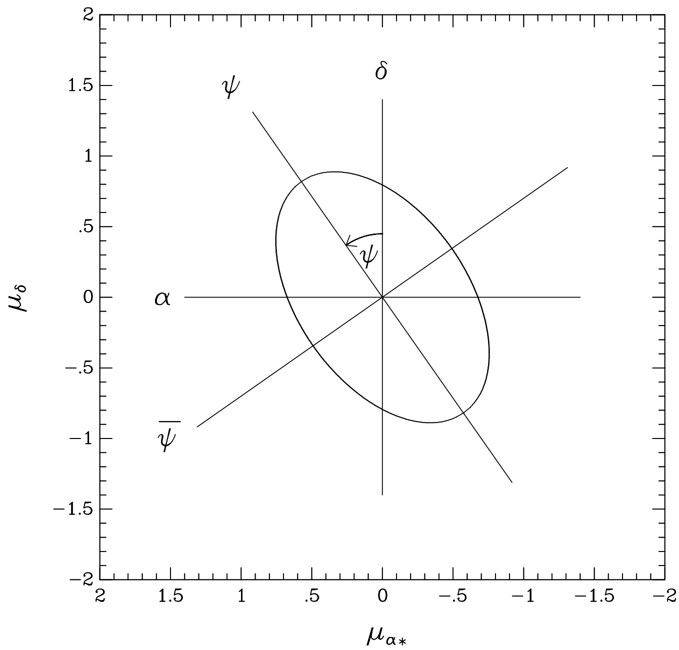

Fig. 2. The error ellipsoid of the measuring errors of $\Delta\mu$ is tilted with respect to the equatorial system (δ, α) by an angle ψ. The major axis of the error ellipsoid points in the direction ψ, the minor axis towards $\overline{\psi}$.

We have also to take into account that the components $\mu_{H,\alpha*}$ and $\mu_{H,\delta}$ of the HIPPARCOS proper motions are correlated. The corresponding correlation coefficient $\rho_{\mu\alpha*,\mu\delta}$ is given in the HIPPARCOS Catalogue. The components of the ground-based proper motions, μ_F or μ_{GC}, are not correlated with μ_H. All the correlations of the components of μ_0 with other quantities are neglected here, since they are usually very small, because the measuring error of the ground-based position $x_F(T_F)$ is always much larger than that of the HIPPARCOS position $x_H(T_H)$. For the same reason, we neglect the cross-correlations between $x_{H,\alpha*}$ and $\mu_{H,\delta}$, and between $x_{H,\delta}$ and $\mu_{H,\alpha*}$. From the correlation coefficient $\rho_{\mu\alpha*,\mu\delta}$, we can derive the covariance γ which is the same for μ_H, $\Delta\mu_{0H}$, and $\Delta\mu_{FH}$:

$$\gamma = \rho_{\mu\alpha*,\mu\delta} \ \varepsilon_{\mu,H,\alpha*} \ \varepsilon_{\mu,H,\delta} \ . \tag{15}$$

We shall now discuss the statistical significance of the proper motion difference $\Delta\mu_{0H}$. The result for this pair of proper motions is, however, directly adaptable for the two other differences, $\Delta\mu_{FH}$ and $\Delta\mu_{0F}$.

We assume that the measuring errors in $\Delta\mu_{0H,\alpha*}$ and $\Delta\mu_{0H,\delta}$ follow Gaussian distributions with mean zero and dispersions $\varepsilon_{\Delta\mu,0H,\alpha*}$ and $\varepsilon_{\Delta\mu,0H,\delta}$. However, if $\gamma \neq 0$, the directions of α and δ are generally not the principal axes of the error ellipsoid. Instead, these principle axes are rotated with respect to the equatorial system by an angle ψ (see Fig. 2). The angle ψ is derived from

$$sin\, 2\psi = 2\gamma \,/\, k \ , \tag{16}$$

$$cos\, 2\psi = -\left(\varepsilon^2_{\Delta\mu,0H,\alpha*} - \varepsilon^2_{\Delta\mu,0H,\delta}\right) \,/\, k \ , \tag{17}$$

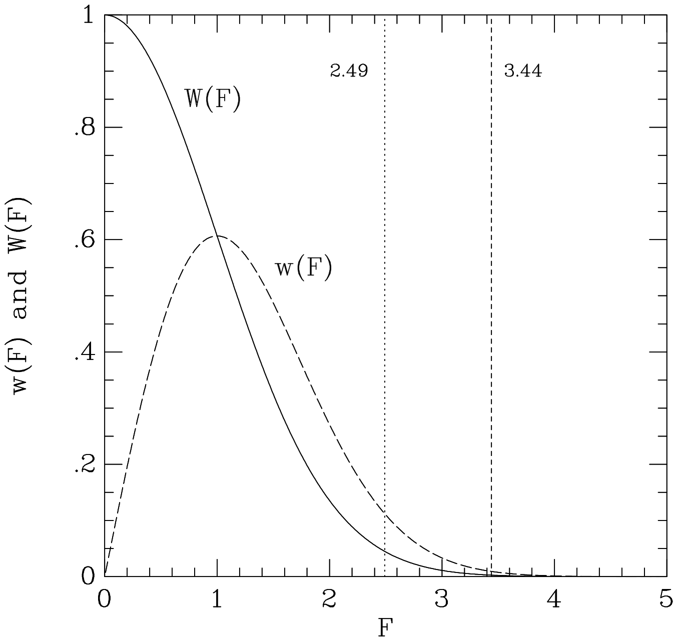

Fig. 3. The function $W(F)$, given by Eq. (24), describes the probability to find by chance an observed value of the test parameter larger than F. The differential probability $w(F)$ is given by Eq. (25). The two adopted critical values, $F > 3.44$ for $\Delta\mu$ binaries and $F < 2.49$ for single-star candidates, are indicated.

with the auxiliary quantity

$$k = + \left(\left(\varepsilon^2_{\Delta\mu,0H,\alpha*} - \varepsilon^2_{\Delta\mu,0H,\delta} \right)^2 + 4\,\gamma^2 \right)^{1/2} \quad . \tag{18}$$

The angle ψ is counted from North towards the East, like a position angle. The dispersions along the principle axes of the error ellipsoid are then given by

$$\varepsilon^2_{\Delta\mu,0H,\psi} \;=\; \frac{1}{2} \left(\varepsilon^2_{\Delta\mu,0H,\alpha*} + \varepsilon^2_{\Delta\mu,0H,\delta} + k \right) \;, \tag{19}$$

$$\varepsilon^2_{\Delta\mu,0H,\overline{\psi}} \;=\; \frac{1}{2} \left(\varepsilon^2_{\Delta\mu,0H,\alpha*} + \varepsilon^2_{\Delta\mu,0H,\delta} - k \right) \;. \tag{20}$$

The components of the observed proper-motion difference $\Delta\mu_{0H}$ in the system of the principle axes of the error ellipsoid (directions ψ and $\overline{\psi}$) are

$$\Delta\mu_{0H,\psi} \;=\; +\Delta\mu_{0H,\alpha*} \sin\,\psi + \Delta\mu_{0H,\delta} \cos\,\psi \;, \tag{21}$$

$$\Delta\mu_{0H,\overline{\psi}} \;=\; +\Delta\mu_{0H,\alpha*} \cos\,\psi - \Delta\mu_{0H,\delta} \sin\,\psi \;. \tag{22}$$

Instead of discussing now the statistical significance of $\Delta\mu_{0H,\psi}$ and $\Delta\mu_{0H,\overline{\psi}}$ separately as two linear problems, it is more suitable to discuss the significance of the vector $\Delta\mu_{0H}$ as a two-dimensional problem. For this purpose, we define the test parameter F_{0H} by

$$F^2_{0H} = \left(\frac{\Delta\mu_{0H,\psi}}{\varepsilon_{\Delta\mu,0H,\psi}} \right)^2 + \left(\frac{\Delta\mu_{0H,\overline{\psi}}}{\varepsilon_{\Delta\mu,0H,\overline{\psi}}} \right)^2 \;. \tag{23}$$

In the 'isotropic' case of the measuring errors (i.e. for $\varepsilon_{\Delta\mu,0H,\alpha*} = \varepsilon_{\Delta\mu,0H,\delta} = \varepsilon_{\Delta\mu,0H,1\mathrm{D}}$ and $\rho_{\mu\alpha*,\mu\delta} = 0$), F would simply be the ratio between the proper-motion difference $\Delta\mu_{0H,tot}$ and its measuring error $\varepsilon_{\Delta\mu,0H,1\mathrm{D}}$.

If the star is not a binary, then the uncorrelated variables $\Delta\mu_{0H,\psi}$ and $\Delta\mu_{0H,\overline{\psi}}$ are expected to follow normal distributions with mean zero and dispersions according to Eqs. (19) and (20). In this case, the probability $W(F)$ to find by chance a value of F_{0H} which is equal to or larger than the observed value (given by Eq. (23)), is

$$W(F) = e^{-F^2_{0H}/2} \;. \tag{24}$$

The differential probability $w(F)\,dF$ to find F_{0H} between F and $F + dF$ is given by

$$w(F) = -\frac{dW(F)}{dF} = F_{0H}\; e^{-F^2_{0H}/2} \;. \tag{25}$$

The function $w(F)$ is plotted in Fig. 3. For small F, $w(F)$ increases linearly with F. The function $w(F)$ reaches a maximum at $F = 1$, and declines rapidly for larger values of F.

We conclude from the run of the function $W(F)$ that a high observed value of F_{0H} is a strong indication for the binary nature of the object under consideration. Which minimal value $F_{lim,b}$ for F_{0H} should be used for our $\Delta\mu$ binaries ? We propose to call those objects $\Delta\mu$ *binaries* for which

$$F_{0H} > F_{lim,b} = 3.44 \tag{26}$$

holds. This gives the same level of significance,

$$W(3.44) = 0.0027 \;, \tag{27}$$

as the often used, two-sided 3σ criterion for a one-dimensional normal distribution. It means that among 10 000 truly single stars, only 27 of them would be wrongly classified by us as $\Delta\mu$ binaries. Our proposed value of 3.44 has also been adopted for the G solutions in the HIPPARCOS Catalogue.

While a large observed value of F_{0H} hints strongly to a binary nature of the object, a small value of F_{0H} makes it rather probable that the star is either single or that its orbital motion is below the level set by the measuring accuracy of $\Delta\mu_{0H}$. We propose to call objects with

$$F_{0H} < F_{lim,s} = 2.49 \tag{28}$$

single-star candidates. This limit corresponds to a 2σ criterion,

$$W(2.49) = 0.0456 \;. \tag{29}$$

From 20 truly single stars, 19 of them would be correctly classified as single-star candidates. One of them would be wrongly dismissed. In general, it is impossible to say how many actual binaries are wrongly classified as single-star candidates. This depends on the measuring accuracy of the proper-motion difference and on the distribution function of the orbital velocities of the binaries. We should therefore strongly emphasize the word 'candidate' in our term 'single-star candidate'.

In the case of the GC, we have only one meaningful test parameter, namely $F_{0(GC)H}$. For the FK5, however, three test parameters are available: F_{FH}, F_{0H}, and F_{0F} . They are, of course, not independent, since the proper-motion differences are related by $\Delta\mu_{FH} = \Delta\mu_{0H} - \Delta\mu_{0F}$. While there are various possibilities to combine the three proper-motion differences into a single test parameter, we propose to call an object a '$\Delta\mu$ binary' if *at least one* F value, either F_{FH} or F_{0H} or F_{0F}, is larger than $F_{lim,b}$. For a 'single-star candidate', *all* the F values should be smaller than $F_{lim,s}$. The reasons for this choice will become clearer by the discussion in the next section.

4. Which types of binaries can be detected ?

The types of binaries which can be detected depend, of course, on the nature and quality of the available astrometric data. We concentrate here on the cases FK5 + HIPPARCOS and GC + HIPPARCOS.

Since HIPPARCOS proper motions and positions are already averages over about 3 years of observations, short-period binaries with orbital periods P below 3 years should not be detectable by our method.

For binaries with medium periods of a few decades, say $P \sim 30$ years, the HIPPARCOS proper motion μ_H is essentially an instantaneous value, while the other proper motions $(\mu_F, \mu_0, \mu_{0(GC)})$ can be considered as mean proper motions, close to the center-of-mass motion of the binary. Such double stars can be detected by their large values of $\Delta\mu_{FH}$ and $\Delta\mu_{0H}$ which fully contain the orbital motion of the binary. The difference $\Delta\mu_{0F}$ between the two *mean* proper motions μ_0 and μ_F should be small for such medium periods.

Double stars with long periods, say with $P \sim 1000$ years, can also be detected by our method, if the measuring accuracy is high enough with respect to the orbital motion of the photocenter. This is sometimes the case for nearby objects. For long-period binaries, all the three proper motions are essentially 'instantaneously' measured values: μ_H at T_H, μ_F at about T_F, and μ_0 at about $T_0 = (T_H + T_F)/2$. The exact epochs of μ_F and μ_0 are uncertain because the values μ_F and $x_F(T_F)$ are derived from many ground-based catalogues which are spread over a long interval of time, e.g. more than 200 years in the case of the basic FK5. On the other hand, in these determinations the more recent catalogues have entered with much higher weights than the old catalogues.

For the long-period binaries, the photo-center moves approximately on a curve of second order in time. Using T_H as the reference epoch in the Taylor serie, we may write:

$$x(T) = x_H(T_H) + \mu_H(T_H)\,(T - T_H) + \frac{1}{2}\,g\,(T - T_H)^2, \quad (30)$$

and

$$\mu(T) = \mu_H(T_H) + g\,(T - T_H) \ , \quad (31)$$

where g is the (constant) acceleration of the photo-center in this coordinate. We obtain then

$$\Delta\mu_{FH} = \mu_F(T_F) - \mu_H(T_H) = -g\,(T_H - T_F) \ . \quad (32)$$

Since

$$\begin{aligned}\mu_0(T_0) \ &= \ \frac{x_H(T_H) - x_F(T_F)}{T_H - T_F} \\ &= \ \mu_H(T_H) - \frac{1}{2}\,g\,(T_H - T_F) \ ,\end{aligned} \quad (33)$$

we find

$$\Delta\mu_{0H} = \mu_0(T_0) - \mu_H(T_H) = -\frac{1}{2}\,g\,(T_H - T_F) \ , \quad (34)$$

and

$$\Delta\mu_{0F} = \mu_0(T_0) - \mu_F(T_F) = +\frac{1}{2}\,g\,(T_H - T_F) \ . \quad (35)$$

Hence

$$\Delta\mu_{FH} = 2\,\Delta\mu_{0H} = -2\,\Delta\mu_{0F} \ . \quad (36)$$

This means that $\Delta\mu_{FH}$ is most sensitive to the binary nature of the object for long-period double stars.

Can we decide on the basis of our observed proper-motion differences whether the binary has a medium period or a long one ? For the GC this is impossible, since we have only one quantity, $\Delta\mu_{0(GC)H}$, available. For the FK5, the situation is better. A long-period binary should fulfill approximately the relation between the three proper-motion differences according to Eq. (36), both in α_* and δ. For a binary of medium period and with a small cosmic error c_x in the HIPPARCOS position $x_H(T_H)$, we expect a small value of $\Delta\mu_{0F}$, so that $\Delta\mu_{0H} \sim \Delta\mu_{FH}$. For some binaries of medium periods, the cosmic error c_x may not be negligible with respect to the measuring error $\varepsilon_{x,F}$ of $x_F(T_F)$. Assuming that μ_F and $x_F(T_F)$ are long-term, mean quantities, we can derive the cosmic error c_x by

$$c_x = \Big((\Delta\mu_{0F})^2 - \varepsilon^2_{\Delta\mu,0F} \Big)^{1/2} (T_H - T_F) \ . \quad (37)$$

In some cases, our method seems to detect also short-period binaries, with $P \sim$ 1 - 3 years. Due to the finite number and the sometimes uneven distribution in time of the HIPPARCOS observations, the HIPPARCOS proper motion μ_H of such short-period binaries can deviate from the mean proper motion (e.g. characterized by μ_F) by a significant amount, even if derived from observations spread over 3 years.

5. The sensitivity of the method

The sensitivity of our method is primarily determined by the astrometric accuracy, measured in milliarcsec (mas)/year. For astrophysical considerations it is more appropriate to translate the proper-motion difference $\Delta\mu$ into a velocity difference Δv, measured in km/s:

$$\Delta v\,[\mathrm{km/s}] = 4.74\,\Delta\mu\,[\mathrm{mas/year}]\,/\,p\,[\mathrm{mas}] \ , \quad (38)$$

where p is the parallax of the star. If the HIPPARCOS parallax is statistically significant (e.g. $p > 3\,\varepsilon_p$), we use this value. For the remaining distant stars, photometric or spectroscopic distances should be preferred.

In the following general discussion, we neglect for simplicity the anisotropy of the measuring accuracy: We replace the measuring errors of the proper motions in α_* and δ by a common value $\varepsilon_{\mu,1D}$:

$$\varepsilon^2_{\mu,1\mathrm{D}} = \frac{1}{2}\left(\varepsilon^2_{\mu,\alpha*} + \varepsilon^2_{\mu,\delta}\right) \ . \tag{39}$$

The index 1D indicates that this rms value is the mean measuring error in *one d*irection. Any correlation coefficients are also neglected in this section.

Table 1. Error budget of the proper-motion differences of 847 stars from the basic FK5

Quantity	[mas/year]	[km/s] at r = 10 pc	[km/s] at r = 100 pc
$\varepsilon_{\mu,H,1\mathrm{D}}$	0.67	0.032	0.32
$\varepsilon_{\mu,F,1\mathrm{D}}$	0.84	0.040	0.40
$\varepsilon_{\mu,0,1\mathrm{D}}$	0.60	0.028	0.28
$\varepsilon_{\Delta\mu,FH,1\mathrm{D}}$	1.07	0.051	0.51
$\varepsilon_{\Delta\mu,0F,1\mathrm{D}}$	1.03	0.049	0.49
$\varepsilon_{\Delta\mu,0H,1\mathrm{D}}$	0.90	0.043	0.43
3.44 $\varepsilon_{\Delta\mu,FH,1\mathrm{D}}$	3.68	0.174	1.74
3.44 $\varepsilon_{\Delta\mu,0F,1\mathrm{D}}$	3.54	0.168	1.68
3.44 $\varepsilon_{\Delta\mu,0H,1\mathrm{D}}$	3.10	0.147	1.47

Table 2. Error budget of the proper-motion differences of 11 773 stars from the GC

Quantity	[mas/year]	[km/s] at r = 10 pc	[km/s] at r = 100 pc
$\varepsilon_{\mu,H,1\mathrm{D}}$	0.75	0.036	0.36
$\varepsilon_{\mu,0\,(GC),1\mathrm{D}}$	1.43	0.068	0.68
$\varepsilon_{\Delta\mu,0\,(GC)\,H,1\mathrm{D}}$	1.62	0.077	0.77
3.44 $\varepsilon_{\Delta\mu,0\,(GC)\,H,1\mathrm{D}}$	5.57	0.264	2.64

In Tables 1 and 2 we present the error budget for 847 basic FK5 stars and for 11 773 GC stars. The mean errors are rms averages over the individual mean errors of these stars. The error budget of the stars in the FK5 extension lies between that of the basic FK5 and the GC. The mean errors of μ_F and μ_0 contain the uncertainty in the transformation of the FK5 or GC system to the HIPPARCOS system.

We have selected those stars for which the HIPPARCOS Catalogue gives (linear) standard solutions. In other words, these stars are not contained in the 'Double and Multiple Star Annex (DMSA)' of the HIPPARCOS Catalogue, i.e. they have no C, G, O, V, or X solutions. In addition we have excluded stars which are known to be binaries from ground-based measurements. For the GC, we have furthermore excluded stars with large measuring errors in $\Delta\mu_{0(GC)H}$.

The best measured objects are the basic FK5 stars (Tab. 1). The sensitivity of our method, described by $\varepsilon_{\Delta\mu,0H,1\mathrm{D}}$, is about 0.90 mas/year. This corresponds to a mean measuring error for the velocity difference of 0.043 km/s for a nearby object at a distance $r = 10$ pc from the Sun. The accuracy of 43 m/s is much better than the accuracy of conventional measurements of radial velocities and comes close to the accuracy of the best modern radial velocities. With respect to very accurate radial-velocity measurements, which cover at present a few years only, our method allows to identify binaries with much longer periods. For stars at $r = 100$ pc, the measuring accuracy of our method, 0.43 km/s, is still comparable to conventional radial-velocity measurements. Here our method has the advantage that this accuracy is also attained for early-type stars for which spectroscopic correlation methods have difficulties because of the small number of spectral lines. For rather distant stars, say at $r = 1$ kpc, our method is not very sensitive, due to a measuring error of more than 4 km/s.

For the much larger number of GC stars (Tab. 2), the sensitivity of our method is lower by a factor of about 2 with respect to the basic FK5 stars. For nearby objects at $r = 10$ pc, however, the measuring accuracy of 77 m/s for GC stars is still impressive. Here, our method has the advantage to reach a higher number of objects than the published radial-velocity surveys.

Since we accept only those objects as $\Delta\mu$ binaries which have an F value larger than 3.44, the velocity difference for such candidates has to be larger than about 0.15 km/s for basic FK5 stars and 0.26 km/s for GC stars at $r = 10$ pc, and correspondingly higher for more distant stars. For nearby objects, say up to 25 pc, these values are still very acceptable.

6. Interpretation of the observed velocity difference

Our method provides primarily a qualitative indication on whether or not an object can be classified as $\Delta\mu$ binary or as a single-star candidate. However, the observed velocity difference for a $\Delta\mu$ binary contains obviously also a quantitative information on the character of this probable binary. Binaries of medium and long periods have to be treated differently.

6.1. Binaries with medium periods

We call periods of the order of a few decades, say $P \sim 30$ years, medium periods. In this case, $\Delta\mu_{FH,tot}$ corresponds to the two-dimensional projection of the instantaneous three-dimensional velocity v_{ph} of the photo-center of the binary with respect to the center-of-mass. For a randomly orientated velocity v_{ph}, we have on average:

$$v_{ph}\,[\mathrm{km/s}] = 4.74\left(\frac{\pi}{4}\right)^{-1} \frac{\Delta\mu_{FH,tot}\,[\mathrm{mas/year}]}{p\,[\mathrm{mas}]} \ . \tag{40}$$

If we would prefer to use the median value of v_{ph} instead of the inverse mean one, we had to replace in Eq. (40) $\pi/4 = 0.785$

by $\sqrt{3/4} = 0.866$. In the case of the GC, we use $\Delta\mu_{0(GC)H,tot}$ instead of $\Delta\mu_{FH,tot}$. The velocity of the photo-center, v_{ph}, is related to the instantaneous velocity of component B relative to A, v_{AB}, by

$$v_{ph} = |\tilde{\alpha} - \beta| \, v_{AB} \ , \tag{41}$$

where $\tilde{\alpha}$ and β are the fractions of the mass $\mathcal{M}$ and of the luminosity L of the secondary component B:

$$\tilde{\alpha} = \frac{\mathcal{M}_B}{\mathcal{M}_A + \mathcal{M}_B} \ , \ \beta = \frac{L_B}{L_A + L_B} \ . \tag{42}$$

Usually, we assume that B is dark ($L_B \ll L_A$, $\beta \sim 0$). If desired, a finite value of L_B can be taken into account later by starting an iteration process. For an elliptic orbit of eccentricity e, the time average of v_{AB} is given by

$$v_{AB}\,[\mathrm{km/s}] = 4.74 \, \frac{2\pi\, a\,[\mathrm{AU}]}{P\,[\mathrm{years}]} \, f_v(e) \ , \tag{43}$$

where a is the semi-major axis of the orbit of B relative to A, and

$$f_v(e) = \frac{2}{\pi} \, \boldsymbol{E}\,(e^2) \ . \tag{44}$$

$\boldsymbol{E}(x)$ is the complete elliptic integral of the second kind. We have $f_v(0) = 1$, $f_v(0.5) = 0.93$, and $f_v(1) = 2/\pi = 0.64$. In most of our applications we choose $e = 0.5$ as a typical value. The relation between a and the period P is given by Kepler's third law:

$$(a\,[\mathrm{AU}])^3 \,/\, (P\,[\mathrm{years}])^2 = \mathcal{M}_A\,[\mathcal{M}_\odot] + \mathcal{M}_B\,[\mathcal{M}_\odot] \ . \tag{45}$$

The mass $\mathcal{M}_A$ can be derived from a properly chosen mass-luminosity relation for $\beta \sim 0$. Combining Eqs. (40) - (45), we find

$$\frac{\mathcal{M}_B}{(\mathcal{M}_A + \mathcal{M}_B)^{2/3}} \sim |\tilde{\alpha} - \beta| \, (\mathcal{M}_A + \mathcal{M}_B)^{1/3} = \varphi = (2/\pi^2) \, f_v^{-1}(e) \ (P\,[\mathrm{years}])^{1/3} \, (p\,[\mathrm{mas}])^{-1} \ \Delta\mu_{FH,tot}\,[\mathrm{mas/year}] \ . \tag{46}$$

The quantity φ in Eq. (46) is closely linked to the 'mass function' of spectroscopic binaries which equals $(\varphi \sin i)^3$ for $\beta = 0$, where i is the inclination of the orbit. From Eq. (46) we can determine $\mathcal{M}_B\,[\mathcal{M}_\odot]$ (e.g. iteratively) for a given value of P, say for $P = 30$ years. For $\tilde{\alpha} \ll 1$, $\mathcal{M}_B$ scales with $P^{1/3}$. Having determined a typical value of $\mathcal{M}_B$ from our data, we can use again the mass-luminosity relation for deriving L_B and hence the magnitude difference $m_B - m_A$. This is also a check on our approximation for β ($\beta \sim 0$).

Knowing now $\mathcal{M}_A$ and $\mathcal{M}_B$ we can derive the semi-major axis from Eq. (45). This allows us then to predict the projected separation ρ_{AB} of B and A, and the projected distance ρ_{ph} of the photo-center from the center-of-mass:

$$\rho_{AB}\,[\mathrm{mas}] = \frac{\pi}{4} \, f_r(e) \ a\,[\mathrm{AU}] \ p\,[\mathrm{mas}] \ , \tag{47}$$

with

$$f_r(e) = 1 + \frac{1}{2}\, e^2 \ , \tag{48}$$

and

$$\rho_{ph} = |\tilde{\alpha} - \beta| \, \rho_{AB} \ . \tag{49}$$

Our method allows therefore to predict approximate values of $\mathcal{M}_B$, Δm, ρ_{AB} and ρ_{ph}. Clearly, these estimated values have a large statistical noise, because of the many unknowns (geometry, orbital phase, period, eccentricity). Nevertheless, the values of ρ_{AB} and Δm are interesting for the planning of observations for direct imaging. The value of ρ_{ph} is an estimate of the (two-dimensional) cosmic error in the HIPPARCOS position of this star. We can also calculate the expected *total* variation of the radial velocity of A in time, Δv_{rad}. In a sufficient approximation ($e = 0$, $\beta = 0$), we find

$$\Delta v_{rad}\,[\mathrm{km/s}] \sim 2.6 \ \Delta v_{FH,tot}\,[\mathrm{km/s}] = 2.6 \ \frac{4.74 \, \Delta\mu_{FH,tot}\,[\mathrm{mas/year}]}{p\,[\mathrm{mas}]} \ , \tag{50}$$

This estimate may be helpful for the planning of radial-velocity observations.

6.2. *Long-period binaries*

In the case of long periods, the proper-motion differences do not allow us to determine the orbital *velocity* of the photo-center. Instead we obtain the orbital *acceleration*, g_{ph}, of the photo-center:

$$g_{ph,tot,\mathrm{2D}} = \frac{\Delta\mu_{FH,tot}}{|T_H - T_F|} \ . \tag{51}$$

The components of g_{ph} in α_* and δ provide immediately the position angle of B relative to A, since the vector of g_{ph} points from A to B, if $\tilde{\alpha} - \beta > 0$. On (linear) average, we have for a two-dimensional projection $g_{ph,tot,\mathrm{2D}}$ of a three-dimensional vector g_{ph}:

$$g_{ph,tot,\mathrm{2D}} = \frac{\pi}{4} \ g_{ph,tot,\mathrm{3D}} \ . \tag{52}$$

Similar to Eq. (41), we have

$$g_{ph,tot,\mathrm{3D}}\,[\mathrm{mas/year^2}] = |\tilde{\alpha} - \beta| \, g_{AB}\,[\mathrm{AU/year^2}] \, p\,[\mathrm{mas}], \tag{53}$$

with

$$g_{AB} = a \, \left(\frac{2\pi}{P}\right)^2 \, f_g(e) \tag{54}$$

as an average over the orbital phase, where

$$f_g(e) = (1 - e^2)^{-1/2} \ . \tag{55}$$

Proceeding in the same way as in Sect. 6.1, we derive

$$\frac{\mathcal{M}_B}{(\mathcal{M}_A + \mathcal{M}_B)^{2/3}} \sim |\tilde{\alpha} - \beta| \, (\mathcal{M}_A + \mathcal{M}_B)^{1/3} = \varphi = (1/\pi^3) \, f_g^{-1}(e) \, (P\,[\mathrm{years}])^{4/3} \, (p\,[\mathrm{mas}])^{-1} \ \Delta\mu_{FH,tot}\,[\mathrm{mas/year}] \, (|T_H - T_F|\,[\mathrm{years}])^{-1} \ . \tag{56}$$

We can now determine $\mathcal{M}_B$ from Eq. (56) for a given period P, say for $P = 1000$ years. In the case of long periods, $\mathcal{M}_B$

scales as $P^{4/3}$ for $\tilde{\alpha} \ll 1$. For estimating ρ_{AB} and ρ_{ph}, we use Eqs. (45), (47), and (49). The change in radial velocity *during a time interval* Δt is expected to be (for $\beta = 0$)

$$\Delta v_{rad}\,[\mathrm{km/s}] \sim \Delta v_{FH,tot}\,[\mathrm{km/s}]\,\frac{\Delta t\,[\mathrm{years}]}{|T_H - T_F|\,[\mathrm{years}]} = \frac{4.74\,\Delta\mu_{FH,tot}\,[\mathrm{mas/year}]\,\Delta t\,[\mathrm{years}]}{|T_H - T_F|\,[\mathrm{years}]}\,. \quad (57)$$

Since Eqs. (56) and (46) have a similar structure, we can easily derive the ratio of $\mathcal{M}_{B,long}$ to $\mathcal{M}_{B,medium}$ for $\tilde{\alpha} \ll 1$ and $\beta \sim 0$:

$$\left(\frac{\mathcal{M}_{B,long}}{\mathcal{M}_{B,medium}}\right) = \frac{1}{2\pi}\,\frac{f_v(e)}{f_g(e)}\,\frac{(P_{long}\,[\mathrm{years}])^{4/3}}{(P_{medium}\,[\mathrm{years}])^{1/3}\,(|T_H - T_F|\,[\mathrm{years}])}\,. \quad (58)$$

For $P_{long} = 1000$ years, $P_{medium} = 30$ years, $T_H - T_F = 40$ years, and $e = 0.5$, we find a ratio of about 10.

In some cases we do not find a plausible solution for $\mathcal{M}_A$ and $\mathcal{M}_B$ from Eq.(56) or even from Eq.(46): For $\beta = 0$, we may obtain that the mass $\mathcal{M}_B$ of the secondary is larger than or comparable to the mass $\mathcal{M}_A$ of the primary, which is not very probable. If we allow for $\beta \neq 0$ and assume that both components are main-sequence stars, we find a solution for $\mathcal{M}_A$ and $\mathcal{M}_B$ only if $(\varphi/\mathcal{M}^{1/3}_{A,\beta=0}) \lesssim 0.3$. Of course, we may then change the period P from its assumed standard value to such a lower value that the estimates for $\mathcal{M}_A$ and $\mathcal{M}_B$ become now reasonable.

In Sects. 6.1 and 6.2, we have neglected the difference between the expectation value $\langle q \rangle$ of a quantity q and $1/\langle 1/q \rangle$, and between $\langle q \rangle^n$ and $\langle q^n \rangle$. For most quantities, this difference is smaller than the inherent 'noise' in the expectation value caused by the unknown orbital phase of the star and by the unknown spatial orientation of its orbit, which are treated both in a statistical way only.

7. A few examples

In order to illustrate our method we provide in Tabs. 3 and 4 a few examples. Individual comments on these stars are given in the following subsections. A discussion of other FK and GC objects will follow in subsequent papers.

7.1. HIP 58001 = γ UMa

This is a very good example for a new binary detected by our $\Delta\mu$ method. The high values of F_{FH}, F_{0H}, and $F_{0(GC)H}$ (of the order of 20) leave no doubt on the binary nature of γ UMa. The FK5 and GC values are in perfect agreement. Since F_{0F} is rather small (indicating a good agreement between the 'mean' proper motions μ_F and μ_0), γ UMa is most probably a binary with a medium period P of a few decades. Earlier, not very detailed reports on a variability of the radial velocity of γ UMa have not been confirmed by more recent observations, according to Hubrig and Mathys (1994). Since γ UMa A has a large rotational velocity ($v_{rot} \sin i = 165$ km/s, Abt and Morrell 1995), accurate radial-velocity measurements are difficult. The expected total variation of the radial velocity, about 4 km/s, is so small that it is not astonishing that γ UMa has not been detected as a spectroscopic binary up to now.

For an assumed medium period of $P = 30$ years, we predict in Tab. 4 a separation between the two components of γ UMa of about $0''.5$ and a magnitude difference in V of nearly 10^m. It will certainly be very difficult to observe the secondary component, probably a late dwarf or a white dwarf, by direct methods. A very long period is improbable for γ UMa, since this would lead to $\mathcal{M}_B > \mathcal{M}_A$ in our statistical estimate of $\mathcal{M}_B$ for $\beta = 0$. This confirms our earlier conclusion on P based on the small value of F_{OH}

γ UMa is one of the members of the Ursa Major Star Cluster. By using the quality of the convergence of the FK4 proper motions of 6 members of the UMa cluster, Wielen (1978a, b) has shown that the internal velocity dispersion of this cluster is as small as 0.1 km/s, corresponding to 1 mas/year at the distance of the cluster ($r \sim 25$ pc). The FK4 proper motions are obviously not affected by cosmic errors and represent already very closely 'mean' proper motions. The fact that the FK4 proper motion gave a cluster parallax for γ UMa (39.3 ± 0.3 mas) which is in perfect agreement with the HIPPARCOS trigonometric parallax (39.0 ± 0.7 mas) is a very strong indication that the cosmic error in the HIPPARCOS proper motion of γ UMa is due to a binary motion and is not artificially caused by a wrong estimate of the measuring errors in the ground-based proper motions. In contrast, a cluster parallax of γ UMa derived by using the HIPPARCOS proper motion would be larger than the HIPPARCOS trigonometric parallax p_H by about +5 mas or 8 mean errors of p_H. By a mere accident the cosmic error (about 13 mas/year) in the HIPPARCOS proper motion of γ UMa points nearly exactly towards the convergent point of the UMa cluster. There is certainly no physical reason behind this coincidence. While ζ UMa is a well-known multiple system, none of the remaining 4 FK members of the UMa cluster are $\Delta\mu$ binaries. However, it is also true that none of these 4 FK stars is qualified as a single-star candidate.

7.2. HIP 16537 = ε Eri

This star is an example for a $\Delta\mu$ binary at the verge of detectability and significance. The value $F_{0(GC)H} = 3.96$ is larger than $F_{lim,b} = 3.44$, while $F_{FH} = 3.25$ is slightly smaller than our adopted limit. We measure a velocity discrepancy $\Delta v_{FH,tot}$ of 57 m/s. The fact that $\Delta\mu_{FH} \sim \Delta\mu_{0H}$ while $\Delta\mu_{0F} \sim 0$, favours an orbital period of medium length.

Accurate radial-velocity measurements by Walker et al. (1995) suggest a period of $P = 9.88$ years and a radial-velocity amplitude of about 15 m/s. The significance of this result is disputed by the authors themselves (Walker et al. 1995) and by others (see Greaves et al. 1998). If the radial-velocity amplitude of 15 m/s and our value of $\Delta v_{FH,tot}$ (57 m/s) are correct, then the orbital inclination i must be quite small, of the order of 10 - 20°. This would be in agreement with the orientation of

Table 3. Some examples od $\Delta\mu$ binaries (γ UMa, ε Eri, ι Vir) and of single-star candidates (47 UMa, δ Pav)

	HIP-No.:	58001		16537		69701		53721		99240	
	FK-No.:	447		127		525		1282		754	
	Name:	γ UMa		ε Eri		ι Vir		47 UMa		δ Pav	
Quantity	Unit										
p_H	[mas]	38.99	± 0.68	310.75	± 0.85	46.74	± 0.87	71.04	± 0.66	163.73	± 0.65
r	[pc]	25.65	± 0.45	3.22	± 0.01	21.39	± 0.40	14.08	± 0.13	6.11	± 0.02
$\varepsilon_{\mu,H,\alpha*}$	[mas/yr]		± 0.48		± 0.98		± 0.91		± 0.58		± 0.55
$\varepsilon_{\mu,H,\delta}$	[mas/yr]		± 0.50		± 0.91		± 0.68		± 0.54		± 0.47
$\varepsilon_{\mu,F,\alpha*}$	[mas/yr]		± 0.47		± 0.45		± 0.53		± 0.62		± 0.98
$\varepsilon_{\mu,F,\delta}$	[mas/yr]		± 0.44		± 0.52		± 0.59		± 0.71		± 1.06
$\varepsilon_{\mu,0,\alpha*}$	[mas/yr]		± 0.35		± 0.30		± 0.33		± 0.45		± 0.80
$\varepsilon_{\mu,0,\delta}$	[mas/yr]		± 0.29		± 0.29		± 0.32		± 0.48		± 0.78
$\varepsilon_{\mu,0\,(GC),\alpha*}$	[mas/yr]		± 0.49		± 0.33		± 0.37		± 0.56		± 0.82
$\varepsilon_{\mu,0\,(GC),\delta}$	[mas/yr]		± 0.40		± 0.32		± 0.36		± 0.53		± 0.84
$\Delta\mu_{FH,\alpha*}$	[mas/yr]	– 12.94	± 0.67	+ 1.70	± 1.08	+ 21.05	± 1.05	– 1.73	± 0.85	+ 0.71	± 1.12
$\Delta\mu_{FH,\delta}$	[mas/yr]	– 1.36	± 0.67	+ 3.31	± 1.05	– 12.21	± 0.90	+ 0.85	± 0.89	– 0.81	± 1.16
$\Delta\mu_{FH,tot}$	[mas/yr]	13.01	± 0.69	3.72	± 1.16	24.33	± 1.10	1.93	± 0.89	1.08	± 1.17
$\Delta\mu_{0H,\alpha*}$	[mas/yr]	– 12.63	± 0.59	+ 1.76	± 1.02	+ 10.52	± 0.97	– 1.10	± 0.73	+ 0.60	± 0.97
$\Delta\mu_{0H,\delta*}$	[mas/yr]	– 0.81	± 0.58	+ 2.49	± 0.96	– 14.29	± 0.75	– 0.31	± 0.72	– 1.16	± 0.91
$\Delta\mu_{0H,tot}$	[mas/yr]	12.66	± 0.60	3.05	± 1.11	17.74	± 0.95	1.14	± 0.71	1.31	± 0.95
$\Delta\mu_{0F,\alpha*}$	[mas/yr]	+ 0.31	± 0.59	+ 0.06	± 0.54	– 10.53	± 0.62	+ 0.63	± 0.77	– 0.11	± 1.27
$\Delta\mu_{0F,\delta}$	[mas/yr]	+ 0.55	± 0.53	– 0.82	± 0.60	– 2.08	± 0.67	– 1.16	± 0.86	– 0.35	± 1.32
$\Delta\mu_{0F,tot}$	[mas/yr]	0.63	± 0.55	0.82	± 0.60	10.73	± 0.62	1.32	± 0.84	0.37	± 1.32
$\Delta\mu_{0\,(GC)\,H,\alpha*}$	[mas/yr]	– 13.42	± 0.69	+ 1.40	± 1.03	+ 16.24	± 0.98	– 0.58	± 0.81	+ 0.31	± 0.99
$\Delta\mu_{0\,(GC)\,H,\delta}$	[mas/yr]	– 1.92	± 0.64	+ 3.82	± 0.96	– 12.62	± 0.77	+ 0.44	± 0.76	+ 0.31	± 0.96
$\Delta\mu_{0\,(GC)\,H,tot}$	[mas/yr]	13.56	± 0.72	4.07	± 1.06	20.57	± 1.01	0.73	± 0.83	0.44	± 0.95
F_{FH}	[number]	19.80		3.25		22.08		2.16		0.91	
F_{0H}	[number]	22.68		2.78		19.82		1.62		1.38	
F_{0F}	[number]	1.18		1.39		17.19		1.59		0.28	
$F_{0\,(GC)\,H}$	[number]	19.85		3.96		20.60		0.88		0.46	
$\Delta v_{FH,tot}$	[km/s]	1.58	± 0.08	0.057	± 0.018	2.47	± 0.11	0.129	± 0.059	0.031	± 0.034
$\Delta v_{0H,tot}$	[km/s]	1.54	± 0.07	0.047	± 0.017	1.80	± 0.10	0.076	± 0.047	0.038	± 0.028
$\Delta v_{0F,tot}$	[km/s]	0.08	± 0.07	0.013	± 0.009	1.09	± 0.06	0.088	± 0.056	0.011	± 0.038
$\Delta v_{0\,(GC)\,H,tot}$	[km/s]	1.65	± 0.09	0.062	± 0.016	2.09	± 0.10	0.049	± 0.055	0.013	± 0.028

the stellar pole of ε Eri which corresponds to $i \sim 30 \pm 15^\circ$, as deduced by Saar and Osten (1997). Using $i = 15^\circ$ and the data given in Tab. 4, the mass of the secondary component of ε Eri would be about 4 Jupiter masses, corresponding to a massive planet or a low-mass brown dwarf. The semi-major axis of the photo-center of ε Eri would be 6 mas, and the semi-major axis of the orbit of the planet would be about 4 AU or $1''\!.4$.

Blazit et al. (1977) published a speckle measurement of ε Eri, giving a binary separation of 48 ± 5 mas. Later speckle observations did not resolve the star (with upper limits of about 30 - 35 mas). In any case, such a close component would not be detected by our $\Delta\mu$ method, since its orbital period (about 20 days) would be much smaller than the 3 years over which the HIPPARCOS proper motion of ε Eri is averaged.

Gatewood (1998, private communication) found by using the Multichannel Astrometric Photometer (MAP) that any deviation of ε Eri from a linear motion is smaller than about 1.4 mas over a period of time of about eight years. This is in contradiction with our estimate of $a_{ph} \sim 6$ mas, based on $\Delta\mu \sim$ 4 mas/year. Even for a much longer period P, our value of $\Delta\mu$ would imply larger deviations than the upper limit claimed by Gatewood. The reason for this discrepancy is unclear.

Greaves et al. (1998) detected a dust ring around ε Eri by measuring the dust emission at $\lambda = 0.85$ mm. The asymmetries and substructures within this ring can hardly be due the potential planet discussed above, since the ring has a radius of about 30 AU, much larger than the semi-major axis of the planet discussed here ($a \sim 4$ AU). As noted by Greaves et al. (1998) the ε Eri ring system is roughly circular and hence appears close to face-on, in agreement with the small inclination derived from our comparison of $\Delta v_{FH,tot}$ with the radial-velocity amplitude, if we assume co-planarity of the planetary orbit and of the ring.

7.3. HIP 69701 = ι Vir

This star is an example for a probable long-period binary. At least the proper-motion differences in α_* are consistent with our criterion for a long period, given by Eq. (36). The proper-motion differences in δ, however, do not obey Eq. (36). This may indicate that the orbital period of ι Vir is not very long, perhaps of the order of 200 years.

The amplitude of the orbital motion of the photo-center of ι Vir amounts to a few hundred mas at least. This can be seen from the results of modern meridian-circle observations used in the construction of the FK5 (Schwan 1990, unpublished). The unusual behaviour of ι Vir has also been noted by Morrison et al. (1990) in a comparison of recent meridian-circle positions with the FK5 prediction. Our $\Delta\mu$ method fully confirms the earlier suggestion that ι Vir is a double star.

7.4. HIP 53721 = 47 UMa

From radial-velocity measurements, Butler and Marcy (1996) have detected a planet orbiting 47 UMa. The orbital period is 3.0 years and the radial-velocity amplitude is 48 m/s. The direct positional measurements by HIPPARCOS are not accurate enough to confirm the orbital motion of 47 UMa due to this planet (Perryman et al. 1996). Our $\Delta\mu$ method is also not able to detect such a planet since its orbital period is too short (the HIPPARCOS proper motion is averaged over 3 years) and the velocity amplitude is rather small (our measuring error of Δv is of the order of 60 m/s for 47 UMa). Our $\Delta\mu$ method indicates strongly, however, that 47 UMa is otherwise a single-star candidate, i.e. that no massive companion of 47 UMa exists, since all the F values of 47 UMa are below our limit of 2.49. The largest velocity discrepancy is $\Delta v_{FH,tot}$ = 129 ± 62 m/s, i.e. not significantly different from zero. This is confirmed by the radial-velocity observations which do not show any systematic trend in time (beside the 3-years period). In summary, 47 UMa seems to be a *single star*, but with at least one planet.

7.5. HIP 99240 = δ Pav

This is an example for a good single-star candidate. All its F values are rather small, below 1.4 in each case. The measuring error of the total velocity discrepancy Δv is about 30 m/s, and hence comparable to that of modern radial-velocity measurements. The published radial velocities do not show any significant variations in time. This excludes massive secondaries with periods below the period limit of our $\Delta\mu$ method (i.e. a few years), if the orbit is not nearly face-on. A low-mass secondary with a short period cannot be ruled out. A very long-period companion can be probably excluded, because its separation of many seconds of arc would have allowed its visual detection, if the magnitude difference is not extremely large.

Table 4. Estimated values of the masses, magnitude differences, and separations for three examples of $\Delta\mu$ binaries

	HIP-No.:	58001	16537	69701
	FK-No.:	447	127	525
	Name:	γ UMa	ε Eri	ι Vir
Quantity	Unit			
$m_{V,tot}$	[mag]	2.41	3.72	4.07
$M_{V,tot}$	[mag]	+ 0.36	+ 6.18	+ 2.42
$\mathcal{M}_A$ (β=0)	[$\mathcal{M}_\odot$]	3.07	0.85	1.64
Standard medium period $P = 30$ years and $e = 0.5$ assumed:				
φ	[$\mathcal{M}_\odot^{1/3}$]	0.23	0.0081	0.35
$\mathcal{M}_A$	[$\mathcal{M}_\odot$]	3.07	0.85	1.64
$\mathcal{M}_B$	[$\mathcal{M}_\odot$]	0.53	0.0073	0.61
Δm_{AB}	[mag]	9.4	large	6.2
ρ_{AB}	[mas]	518	2560	531
ρ_{ph}	[mas]	76	22	141
Δv_{rad}	[km/s]	4.1	0.15	6.4
Standard long period $P = 1000$ years and $e = 0.5$ assumed:				
φ	[$\mathcal{M}_\odot^{1/3}$]	1.78	0.062	2.80
$\mathcal{M}_A$ (β=0)	[$\mathcal{M}_\odot$]	(3.07)	0.85	(1.64)
$\mathcal{M}_B$ (β=0)	[$\mathcal{M}_\odot$]	(9.70)	0.06	(24.91)
Δm_{AB}	[mag]		large	
ρ_{AB}	[mas]		27033	
ρ_{ph}	[mas]		1754	
Δv_{rad}	[km/s]	$^{(**)}3.0$	$^{(*)}0.009$	$^{(**)}4.8$
A plausible individual solution:				
P	[years]	30.0	obs: 9.88	200
e	[number]	0.5	obs: 0.0	0.5
i	[°]	stat.	15	stat.
φ	[$\mathcal{M}_\odot^{1/3}$]	(see		0.32
$\mathcal{M}_A$	[$\mathcal{M}_\odot$]	above)	0.85	1.64
$\mathcal{M}_B$	[$\mathcal{M}_\odot$]		0.0038	0.55
Δm_{AB}	[mag]		large	6.9
ρ_{AB}	[mas]		a_{AB}: 1355	1865
ρ_{ph}	[mas]		a_{ph}: 6	466
Δv_{rad}	[km/s]		obs: 0.030	$^{(**)}4.8$

obs: Value taken from other observations.
stat: Statistical estimate.
$^{(*)}$: for $\Delta t = 10$ years; $^{(**)}$: for $\Delta t = 100$ years.

8. Conclusions and outlook

We have presented a new method to detect double stars: Our $\Delta\mu$ method is based on a comparison of the HIPPARCOS proper motions with ground-based data provided e.g. by the FK5 or the GC. We therefore call these newly detected double stars '$\Delta\mu$ binaries'.

If the HIPPARCOS proper motion of a star is in good agreement with the ground-based data and if no other indications for a binary nature of the object exist, then we classify the star as a 'single-star candidate'.

For nearby stars, our $\Delta\mu$ method is very sensitive: At a distance of e.g. 10 pc, our measuring accuracy of orbital velocities is of the order of 50 m/s for basic FK5 stars, and of 80 m/s for many GC stars.

For the detected $\Delta\mu$ binaries we obtain statistical estimates for the separation ρ and for the magnitude difference Δm between the components, based on adopted orbital periods P.

Our statistical estimates on the parameters of a $\Delta\mu$ binary would be significantly improved if accurate radial-velocity measurements would at least provide the acceleration component $g_{rad,A}$ from the linear change in time of the radial velocity v_{rad} of the double-star component A. Of course, such a change would also be a desirable confirmation of the double-star nature of the object.

The sensitivity of and the information provided by the $\Delta\mu$ method would be very much increased if a future astrometric satellite (like the proposed projects GAIA, DIVA, SIM) would reobserve the HIPPARCOS stars. In such a case, a second 'instantaneous' proper motion at the new epoch and an additional 'intermediate' proper motion (based on the two instantaneous positions) would be available. However, even in this case, the long-term proper motions derived by using ground-based data would remain useful for our $\Delta\mu$ method, especially for binaries among the bright stars with longer orbital periods.

In Tabs. 3 and 4 we are giving a few examples of $\Delta\mu$ binaries and of single-star candidates, in order to illustrate our method.

In subsequent papers we shall present the individual results of our $\Delta\mu$ method for all the appropriate FK5 and GC stars. The number of newly detected $\Delta\mu$ binaries among these stars is more than one thousand. The fraction is highest among the 1535 basic FK5 stars: about 10 percent of them are newly discovered $\Delta\mu$ binaries.

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