THE COMBINATION OF HIPPARCOS DATA WITH GROUND-BASED ASTROMETRIC MEASUREMENTS

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Abstract. The combination of HIPPARCOS measurements with suitable ground-based astrometric data improves significantly the accuracy of the proper motions of bright stars. The comparison of both types of data allows us also to identify and to eliminate, at least partially, cosmic errors in the quasi-instantaneously measured HIPPARCOS data which are caused by undetected astrometric binaries. We describe a simple averaging method for the combination of two independent compilation catalogues. The combination of the basic FK5 with HIPPARCOS leads to the Sixth Catalogue of Fundamental Stars (FK6). The accuracy of the FK6 proper motions is higher than that of HIPPARCOS by a factor of about 2 in the single-star mode, and by a factor of more than 4 in the long-term prediction mode which takes cosmic errors into account. We present also the error budget for a combination of the Boss General Catalogue (GC) with HIPPARCOS data. We point out problems with known binaries, and identify an ensemble of 'astrometrically excellent stars'.

Key words: Astrometric catalogues – HIPPARCOS – FK6 – GC – binaries – high-precision astrometry

1. Introduction

The HIPPARCOS astrometry satellite has provided very accurate positions, proper motions, and parallaxes for about 118 000 stars (ESA 1997). Many people may therefore be inclined to think that, at least for the stars observed by HIPPARCOS, the ground-based observations, collected over more than two centuries, are of no use anymore. It is correct that the systematic accuracy of the HIPPARCOS data is so high that the system represented by the HIPPARCOS positions and proper motions (ICRS) is superseding the earlier system (FK5), derived from ground-based measurements. However, the random ('individual') accuracy of the HIPPARCOS data, especially of the proper motions, can nevertheless be significantly improved and checked by ground-based astrometric measurements. Why is that so ? The main reasons are: (1) The lower accuracy of older ground-based measurements of stellar positions is often compensated by the large spread in the epochs of observations. This allows us to derive very accurate proper motions from the ground-based data alone or from a combination with HIPPARCOS results (Wielen 1988). (2) Due to undetected astrometric binaries, the HIPPARCOS proper motions can deviate significantly from the more typical proper motions of the center-of-mass (Wielen 1995b, 1997, Wielen et al. 1997). HIPPARCOS proper motions are derived essentially as 'instantaneous' proper motions from data collected within about 3 years only. In contrast, the ground-based observations cover often more than two centuries. The derived proper motions are then already rather close to 'time-averaged' or 'mean' proper motions. Hence the ground-based results allow often to identify and to correct 'cosmic errors' in the instantaneously measured HIPPARCOS proper motions.

2. A Method of Combination

In this section, we shall describe a simple averaging method for combining the HIPPARCOS data (ESA 1997) with that of a ground-based compilation catalogue, such as the FK5 (Fricke et al. 1988) or the GC (Boss et al. 1937). The method neglects some of the correlations between the HIPPARCOS results for a given star, namely the correlations between the results for α and δ and with the parallax. The advantage of the simple averaging method is its transparency which allows easily to grasp the basic advantages and problems in combining two (or more) catalogues. The results for the error estimates have been already presented earlier (Wielen 1988).

Our method assumes that the two catalogues to be combined are based on completely independ measurements. This is always fulfilled in combining ground-based measurements with HIPPARCOS data.

Before combining the two catalogues, we have to reduce them to a common system of positions and proper motions. In practice this means at present that we have to reduce the ground-based catalogues to the HIPPARCOS system. This can be done, for example, by using methods such as adopted in the construction of the FK5 (Bien et al. 1977).

We call the position in one coordinate (say in declination δ) x, the corresponding proper motion μ, and the central epoch T. The epoch T is chosen such that x and μ are not correlated. The central epochs T are usually given explicitly in ground-based catalogues. For HIPPARCOS, the individual epoch T (and the corresponding position $x(T)$) can be easily derived from the data given in the HIPPARCOS Catalogue (see e.g. Wielen 1997, p. 374). The mean measuring errors of x and μ are called ε_x and ε_μ. For systematically corrected ground-based catalogues, these mean errors have to include the uncertainty of the applied systematic corrections (e.g. Wielen et al. 1998). For HIPPARCOS we use the index H, for the ground-based catalogue the index F, and for the catalogue obtained by combining the two catalogues the index C.

The main conceptional device for combining the two catalogues is the introduction of a third proper motion μ_0, which is based on the two central positions (Wielen 1988, 1995a):

$$\mu_0 = \frac{x_H(T_H) - x_F(T_F)}{T_H - T_F} \quad . \tag{1}$$

The mean error of μ_0 is given by

$$\varepsilon_{\mu,0} = \frac{(\varepsilon_{x,F}^2 + \varepsilon_{x,H}^2)^{1/2}}{T_F - T_H} \quad . \tag{2}$$

We introduce the following 'weights' w:

$$w_{x,H} = 1/\varepsilon_{x,H}^2 \quad , \tag{3}$$

$$w_{x,F} = 1/\varepsilon_{x,F}^2 \quad , \tag{4}$$

$$w_{\mu,H} = 1/\varepsilon_{\mu,H}^2 \quad , \tag{5}$$

$$w_{\mu,F} = 1/\varepsilon_{\mu,F}^2 \quad , \tag{6}$$

$$w_{\mu,0} = 1/\varepsilon_{\mu,0}^2 \quad . \tag{7}$$

It can be shown that the position x_C, the proper motion μ_C, and the central epoch T_C of the star in the combined catalogue C can be derived as 'weighted averages'. The proof is not given here. The result is based on the reconstruction of the full normal equations for the two unknowns x and μ from the catalogued data $x(T)$, μ, T, ε_x, ε_μ. The method is an extended version of those described by Kopff et al. (1964) or by Eichhorn (1974, Section 3.2.2). We obtain for the weights in the combined catalogue C:

$$w_{x,C} = w_{x,H} + w_{x,F} \quad , \tag{8}$$

$$w_{\mu,C} = w_{\mu,H} + w_{\mu,F} + w_{\mu,0} \quad . \tag{9}$$

The new central epoch T_C is given by

$$T_C = (w_{x,H}\, T_H + w_{x,F}\, T_F)/w_{x,C} \quad . \tag{10}$$

The position x_C at T_C follows as

$$x_C(T_C) = (w_{x,H}\, x_H(T_H) + w_{x,F}\, x_F(T_F))/w_{x,C} \quad . \tag{11}$$

The combined proper motion μ_C is the weighted average of the three proper motions μ_H, μ_F, and μ_0:

$$\mu_C = (w_{\mu,H}\ \mu_H + w_{\mu,F}\ \mu_F + w_{\mu,0}\ \mu_0)/w_{\mu,C} \quad . \tag{12}$$

The mean errors $\varepsilon_{x,C}$ and $\varepsilon_{\mu,C}$ of $x_C(T_C)$ and μ_C are derived from

$$\varepsilon_{x,C} = (1/w_{x,C})^{1/2} \quad , \tag{13}$$

$$\varepsilon_{\mu,C} = (1/w_{\mu,C})^{1/2} \quad . \tag{14}$$

Hence we have now obtained all the desired quantities for the combined catalogue C.

In real applications, such as the FK6 (see Section 3), the proper motion μ_0 is often very accurate and governs the combined proper motion μ_C. For the positions, the accuracy of the HIPPARCOS position is usually by an order of magnitude higher than that of the ground-based position. This leads to $T_C \sim T_H$ and $x_C(T_C) \sim x_H(T_H)$. Nevertheless, for the predictions of positions at epochs which are not near to T_H, the accuracy of μ_C governs the accuracy of the predicted position. Hence the improved accuracy of μ_C with respect to μ_H is very important for such a prediction of positions.

We should mention that our averaging method has also an interesting 'inverse' application: If we know the combined catalogue C and one of the two underlying catalogues, we can derive back the second underlying catalogue. We have carried out this 'decomposition of a combined catalogue' for the fundamental catalogues FK5, FK4, and FK3. In these cases, the catalogue containing the 'old' observations, e.g. the FK3 in the case of FK4, is published and hence easily available. In contrast, the summary of the 'new' observations (in our example those between the closing dates of FK3 and FK4) is not documented and hence unavailable. One can use the resulting 'pieces' of the FK5 (say FK5-FK4, FK4-FK3, FK3-NFK, NFK) for testing the linearity of the motion of a star or for deriving a non-linear motion (similar to the G solutions of HIPPARCOS). Clearly it would be more accurate to use all the hundreds of individual observational catalogues for this purpose. This is actually our intention for the future, but the decomposition method gives quite rapidly first approximate results.

In the actual combination of a ground-based catalogue with the HIPPARCOS data, we do not use the simple averaging method described above, but a strict method. This strict method uses essentially the normal equations of rank 5 (two positions: α, δ; two proper motions: μ_α, μ_δ; one parallax: p), which can be derived from the HIPPARCOS Catalogue and the ground-based catalogue. The strict method allows for correlations between all the astrometric parameters in a catalogue. At first, it may be astonishing that the ground-based data can influence the resulting parallax. This is due to the fact that the ground-based data improve directly the proper motion. The improved proper motion leads then indirectly to an improved value of the parallax from the HIPPARCOS data. In reality, the correlation between the parallax and the other astrometric parameters is usually so small for the HIPPARCOS data that the improvement of the parallax in the combined catalogue with respect to that derived from HIPPARCOS alone is negligible in most cases. If the star is not single but indeed an undetected astrometric binary, then the combination method leads to a less accurate parallax. In such a case the 'instantaneous' proper motion and position at T_H, not their 'mean' values, have to be used in deriving the parallax from the satellite observations, as it was correctly done in the HIPPARCOS data reduction procedure.

3. Cosmic Errors and Statistical Astrometry

The method of combining two catalogues described above is strictly valid for truly single stars only, because the method assumes that a star moves linearly in time on a straight line in space. Hence we call the result of the direct use of the combination method the 'single-star mode'.

In reality, we can form an ensemble of 'apparently single stars' only. Such an ensemble is usually spoiled by undetected astrometric binaries. In such an ensemble the quasi-instantaneously measured HIPPARCOS proper motions and positions suffer from 'cosmic errors' (Wielen 1995b, 1997). The effect of these errors on the combination of two catalogues can be properly treated by the methods of statistical astrometry. The amount of the cosmic errors in the HIPPARCOS proper motions has been determined by a comparison with the FK5 and checked partially with the GC (Wielen 1995b, Wielen et al. 1997, 1998). For example, averaged over about 1200 FK5 stars, we obtain a cosmic error in μ_H of $c_\mu = 2.13$ mas/year, and $c_x \sim 10$ mas in x_H. The cosmic error c_μ can be calibrated as a function of the stellar distance r (obtained usually from the HIPPARCOS parallaxes). The derived cosmic errors in μ_H and x_H are usually larger than the measuring errors $\varepsilon_{\mu,H}$ and $\varepsilon_{x,H}$, and are therefore quite significant.

The 'best prediction' of stellar positions based on a combination of two catalogues affected by cosmic errors can be derived in general by applying the concepts of statistical astrometry (Wielen 1997). At present, we use some simplifying assumptions, because we lack the observational data for determining the set of correlation functions needed for a full treatment. We assume that the ground-based catalogue (e.g. the FK5 or the GC) is a fully time-averaged, mean catalogue, without any cosmic errors left. This assumption is certainly not strictly valid, but it should give already quite reasonable results.

Using the cosmic errors in μ_H and x_H, we can derive two other types of predictions for the positions of the stars: (1) The 'long-term prediction mode': This prediction should be used for epochs T which differ from $T_H =$ 1991.25 by more than about 10 years. We obtain the long-term average mode by replacing $\varepsilon^2_{\mu,H}$ by $\varepsilon^2_{\mu,H} + c^2_\mu$, and $\varepsilon^2_{x,H}$ by $\varepsilon^2_{x,H} + c^2_x$. Otherwise, the method of combining the two catalogues remains the same. (2) The 'short-term prediction mode': The cosmic errors are now added to the measuring errors of the 'mean' catalogue, i.e. $\varepsilon^2_{\mu,F}$ is replaced by $\varepsilon^2_{\mu,F} + c^2_\mu$ and $\varepsilon^2_{x,F}$ by $\varepsilon^2_{x,F} + c^2_x$, while $\varepsilon_{\mu,H}$ and $\varepsilon_{x,H}$ are not changed. The short-term prediction is valid for epochs which differ from T_H by a few years only. For a smooth transition from the short-term to the long-term prediction, we would have to know the proper correlation functions (see Wielen 1997).

TABLE I
Error budget for FK6 proper motions in the 'single-star mode'

Typical mean errors of proper motions
(in one component, averaged over $\mu_{\alpha*}$ and μ_δ; units: mas/year)

	rms average	median
HIPPARCOS	0.82	0.63
FK5		
random	0.76	0.64
system	0.28	0.24
total	0.81	0.70
μ_0 (total)	0.58	0.49
FK6	0.35	0.33
ratio of HIPPARCOS to FK6 errors	2.3	1.9

4. The Sixth Catalogue of Fundamental Stars (FK6)

The FK6 is derived by combining the ground-based data summarized in the FK5 (Part I, Fricke et al. 1988) with the HIPPARCOS measurements (ESA 1997). A more detailed description of the construction of the FK6 is given in another paper (Wielen et al. 1998).

In Table I, we give the formal error budget of the FK6 for all the basic fundamental stars in the single-star mode. Table II presents the error budget of the FK6 for 1202 stars in the long-term prediction mode. The results of the short-term prediction mode differ only slightly from that of the direct use of the HIPPARCOS data and are not given here. We should warn those readers who would like to test our numerical results quoted for the FK6 by using the other quantities of Tables I and II as input data. The method of combination works star by star, while the tables give ensemble averages. Since $1/<\varepsilon^2>$ is in general not exactly equal to $<1/\varepsilon^2>$, such a test must slightly fail.

How much better are the FK6 proper motions than the direct HIPPARCOS proper motions ? The gain in accuracy in the single-star mode is about a factor of 2, and in the long-term prediction mode a factor of more than 4. These improvements are certainly significant and favour strongly the use of the FK6.

TABLE II
Error budget for FK6 proper motions in the 'long-term prediction mode'

Typical mean errors of proper motions
(in one component, averaged over $\mu_{\alpha *}$ and μ_δ; units: mas/year)

	rms average	median
HIPPARCOS		
measuring error	0.68	0.60
cosmic error	2.13	(2.04)
total	2.24	(2.13)
FK5		
random	0.77	0.66
system	0.28	0.24
total	0.82	0.70
μ_0 (total)	0.59	0.51
FK6	0.49	0.45
ratio of HIPPARCOS (total) to FK6 errors	4.6	(4.7)

5. Combination of the GC with HIPPARCOS data

For the combination with the very accurate HIPPARCOS data, old ground-based observations are especially valuable, because they have large epoch differences with respect to T_H and often still a reasonable accuracy. A carefully compiled catalogue of old observations of more than 30 000 stars is the GC (Boss et al. 1937). Therefore we have applied our combination method also to the GC. We call the resulting combined catalogue GC+HIP.

In Table III we present the formal error budget of the GC+HIP in the single-star mode. Table IV gives the error budget of the GC+HIP in the long-term prediction mode. For the cosmic errors c_μ and c_x in the HIPPARCOS proper motions and positions of the GC stars, we use the results determined from the basic FK5 stars. This is certainly an approximation only: The GC stars are in general fainter in apparent magnitude than the basic FK5 stars (by about 1.9 magnitudes), and they are more distant (median parallax of GC stars: 6.8 mas, of basic FK5 stars: 11.3 mas). Nevertheless, the comparison of μ_0 of the GC+HIP with μ_H for GC stars has essentially lead to the same values of c_μ and c_x as for the basic FK5 stars (Wielen et al. 1998).

How important the old observations are, is impressively shown by the combination of the GC data with the HIPPARCOS measurements for 1534

TABLE III
Error budget for GC+HIP proper motions in the 'single-star mode'

Typical mean errors of proper motions
(in one component, averaged over $\mu_{\alpha*}$ and μ_δ; units: mas/year)

Sample of stars:	29 717 GC		11 773 GC		1534 FK from GC	
	rms av.	median	rms av.	median	rms av.	median
HIPPARCOS	1.47	0.72	0.76	0.68	0.82	0.63
GC (total)	10.58	9.29	8.60	7.45	3.36	2.39
μ_0 (total)	1.79	1.73	1.43	1.44	0.66	0.55
GC+HIP	0.73	0.63	0.62	0.57	0.42	0.39
ratio of HIPPARCOS to GC+HIP errors	2.0	1.14	1.23	1.19	2.0	1.6

basic FK stars. From a comparison of the last columns of Table III with Table I we see that for these FK stars the combined catalogue GC+HIP is only slightly less accurate than the FK6 (= FK5+HIP) itself.

The first columns of Table III give the formal error budget for all the 29 717 GC stars which we have identified in the HIPPARCOS Catalogue. In the columns in the middle of Table III, we give the results for 11 773 'apparently single' GC stars. This ensemble contains only those GC stars which have no special HIPPARCOS solutions in the DMSA, are no 'suspected' binaries according to HIPPARCOS, do not occur in the CCDM, and have mean errors of $\mu_{0,\alpha*}$ and $\mu_{0,\delta}$ smaller than 2 mas/year.

In the single-star mode (Table III), the rms average for all the GC stars indicates a significant gain by a factor of 2 with respect to HIPPARCOS, while the median shows a smaller improvement (by 14 percent) for these stars. For the 11 773 GC stars (apparently single and with an accurate μ_0) the gain is about 20 percent. The gain is much higher in the long-term prediction mode (Table IV). For the 11 773 GC stars, the accuracy in the GC+HIP proper motion is by nearly a factor of 2 higher than for the HIPPARCOS proper motion (if we now include the cosmic error c_μ of μ_H).

In individual comparison of μ_0 derived from the GC and HIPPARCOS with a typical accuracy of 2 mas/year, with μ_H with a measuring error of about 1 mas/year, allows us to identify GC stars with large cosmic errors

TABLE IV

Error budget for GC+HIP proper motions in the 'long-term prediction mode'

Typical mean errors of proper motions
(in one component, averaged over $\mu_{\alpha *}$ and μ_δ; units: mas/year)

Sample of stars:	11 773 GC		1201 FK from GC	
	rms average	median	rms average	median
HIPPARCOS				
measuring error	0.76	0.68	0.68	0.60
cosmic error	1.80	(1.66)	2.13	(2.04)
total	1.95	(1.79)	2.24	(2.13)
GC (total)	8.60	7.45	3.47	2.47
μ_0 (total)	1.43	1.44	0.67	0.57
GC+HIP	1.07	1.05	0.61	0.55
ratio of HIPPARCOS (total) to GC+HIP errors	1.8	(1.7)	3.7	(3.9)

(say above 6 mas/year) in their instantaneously measured HIPPARCOS proper motions. These stars are good candidates for being checked by other observational techniques for their probable binary nature.

6. Known Binaries and Astrometrically Excellent Stars

Known binaries require usually a rather individual treatment in the process of combining ground-based data with the HIPPARCOS measurements. The main problem is to find a common 'reference point' for using the method of combination described in Section 2. Such reference points may be the center-of-mass, an instantaneous photo-center, a time-averaged photo-center, the separated components A and B of a double star, etc. Often we have to use statistical methods for 'harmonizing' the reference points of the ground-based data and of the satellite measurements. A more detailed description of the problems with double stars is given by Wielen et al. (1998).

Even after doing the best you can do, the accuracy of the position and of the proper motion of a binary is often lower by an order of magnitude or more compared to single stars. In order to help the user who is interested in stars of highest astrometric accuracy, we are flagging in the FK6 and in the

GC+HIP those stars which behave like well-measured single stars. We call this sample 'astrometrically excellent stars'. These stars should not show any known disturbing duplicity. The measuring accuracy of their positions and proper motions should be good. But most important, their proper motions μ_H and μ_0, and, in the case of the FK6, μ_{FK5} should agree within the measuring errors. This agreement indicates, at least statistically, that the individual cosmic error of μ_H is rather small for these stars.

7. Conclusions

The combination of HIPPARCOS measurements with ground-based astrometric data allows us to improve the accuracy of the proper motions of thousands of stars. Improved data for the basic fundamental stars are presented in the FK6. For many other stars for which the old observations are summarized in the GC, improved data have also been derived. The ground-based data are especially valuable for identifying large individual cosmic errors in the quasi-instantaneously measured HIPPARCOS proper motions of undetected binaries.

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