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Searching for VIMs: An astrometric method to detect the binary nature of double stars with a variable component

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Abstract. The photo-center of a close double star in which one component has a variable luminosity moves on the sky in the same rhythm as the total luminosity of the system varies in time. This provides an astrometric method to detect the binary nature of the system if more direct methods fail. We propose to call such systems 'variability-induced movers', or VIMs. We discuss the required properties of a system for being detected as a VIM, such as separation, light amplitude, and relative brightness of the components. High-precision astrometric instruments like the HIPPARCOS satellite or the proposed mission GAIA should be able to detect a significant number of VIMs among the variables. In a proper astrometric treatment of variables, all the observed variable stars should be tested for beings VIMs and treated as such if necessary. We present a quantitative method to handle VIMs by means of the available astrometric and photometric observations carried out by HIPPARCOS. Different schemes can be applied to various cases, e.g. for binaries with a fixed relative geometry or for astrometric binaries with orbital motions.

If a close binary has (constant or variable) components of different colour and if the observations are carried out in at least two photometric pass-bands, then the position (and sometimes also the proper motion) of the photo-center of this double star depends on the wavelength, i.e. on the pass-band. Objects with such 'colour-induced displacements' of their photo-centers may be called 'CIDs'. A method to detect such binaries and to treat CIDs is outlined.

Key words: Astrometry – binaries: general – binaries: close – Stars: variables

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1. Introduction

We describe an astrometric method to detect the binary nature of double stars in which one component is a variable star and which have not been discovered already by other methods (e.g. as visual binaries, spectroscopic binaries, eclipsing binaries etc.).

As the measuring instrument we envisage primarily the ESA Astrometry Satellite HIPPARCOS (e.g. Perryman et al. 1989). The method will be, however, even more important for future astrometric space missions with higher accuracy, such as the GAIA project (Lindegren and Perryman 1995). HIPPARCOS is able to detect and to measure visual binaries by various methods rather directly, down to a separation d between the two components of about $d \geq 0.''1 = 100$ mas, if the magnitude difference Δm between the components is not too large (Mignard et al. 1989). Closer double stars (or those with a large Δm) can be detected by the non-linear orbital motion of the photo-center of the binary with respect to its center-of-mass (e.g. Wielen 1995). Here we describe a third method by which HIPPARCOS is able to detect the binary nature of an apparently single star in the case that the luminosity of (at least) one component of this double star varies with time.

The basic idea of the method is to realize that the photo-center of a binary with a variable component moves back and forth on the connecting line between the two components in the same rhythm in which the luminosity of the variable component changes with time. We propose to call such an object a 'variability-induced mover', abbreviated as VIM. This should help to distinguish conceptionally VIMs from ordinary astrometric binaries in which the photo-center moves on a Keplerian orbit around the center-of-mass.

Even if the binary nature of a VIM is already known from other methods, the special treatment of VIMs in the data reduction of missions like HIPPARCOS should avoid otherwise unexplained deviations from the assumed model, e.g. from a linear motion in the standard solution.

In Section 2, we shall describe the VIM method in detail for the simplest case in which the two components do not show any orbital motion with respect to each other. In Section 3, we will generalize the method to binaries with orbital motions.

2. Binaries with fixed relative geometry

We start here with the simplest case of a binary in which the relative positions of the two components do not change significantly during the observational period of the astrometric mission (i.e. over about 3 years for HIPPARCOS). This case is not only simple and educative, but probably also the most frequently found type in practice, at least for HIPPARCOS.

We adopt rectangular coordinates x and y on the sky. The coordinates x and y should represent locally equatorial coordinates, $\alpha\cos\delta$ and δ, or ecliptical coordinates, $\lambda\cos\beta$ and β. The coordinate differences d_x and d_y between the two components are

$$d_x = x_2 - x_1 \ , \tag{1}$$

$$d_y = y_2 - y_2 \ . \tag{2}$$

By assumption, d_x and d_y do not vary with time and represent the fixed relative geometry of the binary. For writing economy we neglect the common proper motions and parallaxes of the two stars. This means that the positions of both stars (x_i, y_i, with $i = 1, 2$) are completely fixed. In real applications, the effects of the proper motion and of the parallax have to be added to x_i and y_i. The separation d between the two components is given by

$$d = \left(d_x^2 + d_y^2\right)^{1/2} \ . \tag{3}$$

We now assume that star 1 is a variable star and that star 2 has a constant luminosity. The variable flux (i.e. the apparent luminosity) of star 1 is called $F_1(t)$; star 2 produces the constant flux F_2. HIPPARCOS measures the combined total flux $F(t)$ of the system:

$$F(t) = F_1(t) + F_2 \ . \tag{4}$$

The flux F is connected with the magnitude scale m as usually by

$$m = -2.5 \lg F + \mathrm{const.} \ . \tag{5}$$

The position $x_p(t)$ and $y_p(t)$ of the photo-center of the binary is given by

$$x_p(t) = \frac{F_1(t)x_1 + F_2 x_2}{F(t)} \ , \tag{6}$$

$$y_p(t) = \frac{F_1(t)y_1 + F_2 y_2}{F(t)} \ . \tag{7}$$

Equations (6) and (7) can be rewritten in a more convenient form:

$$x_p(t) = \frac{F_1(t)x_1 + F_2(x_1 + d_x)}{F(t)} = x_1 + \frac{F_2 d_x}{F(t)} \ , \tag{8}$$

$$y_p(t) = y_1 + \frac{F_2 d_y}{F(t)} \ . \tag{9}$$

As $F(t)$ varies (e.g. periodically) with time, $x_p(t)$ and $y_p(t)$ vary in such a way that the photo-center moves (back and forth) on the line connecting the two components, i.e. on a straight line. The photo-center is closest to the variable star 1, if $F_1(t)$ and hence $F(t)$ is at its maximum, and reaches the largest distance from star 1 at the minimum of $F_1(t)$ and $F(t)$. Hence the position of the photo-center moves qualitatively in the same rhythm as $F(t)$. This is illustrated in Fig.1 for a simple case.

vfig1.eps

Fig. 1. Illustration of the VIM effect by a simple example. The upper panel shows the total light variation $F(t) = F_1(t) + F_2$ for $F_1(t) = 1 + 0.5 \sin\frac{2\pi}{P}t$ and $F_2 = 1 = $ const. (in arbitrary units). The lower panel shows the relative motion of the photo-center, $\lambda(t)$, defined as the distance of the photo-center from the variable star, measured in units of d, the separation of the two stars.

How large is the effect of the variability-induced motion of the photo-center ? Let us call d_{var} the total displacement of the photo-center between the maximum and the minimum of $F(t)$. From Eqs.(8) and (9), we obtain

$$d_{var} = F_2\, d \left(\frac{1}{F_{min}} - \frac{1}{F_{max}}\right) \ , \tag{10}$$

where F_{min} and F_{max} are the minimum and maximum values of the variable total flux $F(t)$, and $F_{1,min}$ and $F_{1,max}$ are the corresponding extrema of $F_1(t)$. We consider the ratios

$$\lambda_{var} = \frac{d_{var}}{d} \ , \tag{11}$$

$$\phi_{max} = \frac{F_{1,max}}{F_2} \ , \tag{12}$$

$$\phi_{min} = \frac{F_{1,min}}{F_2} \ , \tag{13}$$

and obtain from Eq.(10)

$$\lambda_{var} = \frac{1}{1+\phi_{min}} - \frac{1}{1+\phi_{max}} \ . \tag{14}$$

To give an illustrative example, we consider a binary with $F_{1,max} = 2\ F_2$ and $F_{1,min} = 0.5\ F_2$, and a separation of $d = 30$ mas. The total flux $F(t)$ varies then by a factor of $(2+1)/(0.5+1) = 2$, or by $0\stackrel{m}{.}75$ in m_{tot}. From Eq.(14), we derive $\lambda_{var} = (1/1.5) - (1/3) = 1/3$, or $d_{var} = 10$ mas. Such a total displacement of the photo-center by 10 mas would be easily measurable by HIPPARCOS. In Fig.2, we have plotted λ_{var} as a function of ϕ_{min} and ϕ_{max}.

vfig2.eps

Fig. 2. The total relative displacement $\lambda_{var} = d_{var}/d$ of the photo-center of a VIM as a function of the relative fluxes $\phi_{max} = F_{1,max}/F_2$ and $\phi_{min} = F_{1,min}/F_2$ at the maximum and minimum of star 1 with respect to the constant star 2. The solid curves are labeled by λ_{var}, which is constant along these lines. The lower-right part of the figure is a forbidden area because of $\phi_{max} \geq \phi_{min}$. The cross indicates the location of the example discussed in the text, the open circle that of Fig.1.

In Fig.3, we discuss λ_{var} in more practical terms. Let

$$\begin{aligned} \Delta m_{tot} &= m_{tot,min} - m_{tot,max} \\ &= -2.5 \lg \left(\frac{1+\phi_{min}}{1+\phi_{max}} \right) \end{aligned} \qquad (15)$$

be the total variation in $m_{tot}(t)$ of the binary. Δm_{tot} is a quantity directly measurable by HIPPARCOS. Furthermore, we denote by Δm_2 the magnitude difference between the constant magnitude m_2 of star 2 and the minimum magnitude of m_{tot}:

$$\Delta m_2 = m_2 - m_{tot,min} \quad . \qquad (16)$$

If Δm_2 were 0, this would mean that star 1 is completely dark at its minimum. If star 1 reaches at its minimum the same luminosity as star 2, then we would have $\Delta m_2 = 0.75$. For $F_{1,min} = 10\ F_2$, we have $\Delta m_2 = 2.60$, and $F_{1,min} = 100\ F_2$ gives $\Delta m_2 = 5.01$. In Fig.3, we have plotted λ_{var} as a function of Δm_{tot} and Δm_2 according to

$$\lambda_{var} = 10^{-0.4 \Delta m_2} (1 - 10^{-0.4 \Delta m_{tot}}) \quad . \qquad (17)$$

Figure 3 shows that for obtaining a large value of λ_{var}, it is important that Δm_2 is not too large, say smaller than about 3^m (corresponding to $F_{1,min}$ of about 10 F_2). λ_{var} does not increase strongly with Δm_{tot} for Δm_{tot} larger than about 2^m (corresponding to F_{max}/F_{min} of about 5).

vfig3.eps

Fig. 3. The total relative displacement $\lambda_{var} = d_{var}/d$ of the photo-center of a VIM as a function of the total variation Δm_{tot} of the integrated magnitude m_{tot} of the system and of the magnitude difference Δm_2 between star 2 and star 1 at its minimum. The solid curves are labeled by λ_{var}, which is constant along these lines. Other symbols as in Fig.2.

There should be many variable stars in binary systems in which d and λ_{var} are large enough in order to produce a total displacement d_{var} of a few mas or more, which would be measurable by HIPPARCOS.

We now prepare our method for practical applications. In order to do so, we have first to choose a certain total flux $\overline{F}$, or the corresponding magnitude $\overline{m}$, as a 'reference flux' or 'reference brightness' for the binary under consideration. The choice of $\overline{F}$ is arbitrary in principle. In practice, it is useful to choose $\overline{F}$ roughly equal to the mean total flux $< F(t) >$, averaged over time (e.g. over the period of the variable star). The implications of the choice of $\overline{F}$ will become evident later. The position $\overline{x}_p$ of the photo-center for the chosen reference flux $\overline{F}$ follows from Eq.(8):

$$\overline{x}_p = x_1 + \frac{F_2 d_x}{\overline{F}} \quad . \qquad (18)$$

In the following, we shall give usually the equations for the x coordinate only; the corresponding equations for y are always similar and therefore mostly dropped for brevity. The difference $\Delta x_p(t)$ between $x_p(t)$ and $\overline{x}_p$, using Eqs.(8) and (18), is given by:

$$\Delta x_p(t) = x_p(t) - \overline{x}_p = F_2 d_x \left(\frac{1}{F(t)} - \frac{1}{\overline{F}} \right) \quad . \qquad (19)$$

We introduce the function

$$f(t) = \frac{\overline{F}}{F(t)} - 1 = 10^{0.4(m_{tot}(t) - \overline{m})} - 1 \quad , \qquad (20)$$

and the quantities

$$D_x = \frac{F_2}{\overline{F}} d_x = 10^{0.4(\overline{m} - m_2)} d_x \quad , \qquad (21)$$

$$D_y = \frac{F_2}{\overline{F}} d_y = 10^{0.4(\overline{m} - m_2)} d_y \quad . \qquad (22)$$

The position of the photo-center at time t relative to the photo-center for the reference flux $\overline{F}$ is then given by:

$$\Delta x_p(t) = D_x\, f(t) \quad , \qquad (23)$$

$$\Delta y_p(t) = D_y\, f(t) \quad . \qquad (24)$$

The function $f(t)$ is completely determined by the observable total flux $F(t)$ and by the chosen reference flux $\overline{F}$. Since the quantities $\Delta x_p(t)$ and $\Delta y_p(t)$ are also observable quantities, at least in principle, we can derive the quantities D_x and D_y by a least-squares solution, which is even linear in D_x and D_y and therefore immediately convergent.

In the case of HIPPARCOS, the basic observational data are not two-dimensional positions x and y, but one-dimensional abscissae along the so-called reference great circles (RGCs, see e.g. Lindegren and Kovalevsky 1989). Therefore, one has to project the two-dimensional displacements of the photo-center, described by $\Delta x_p(t)$ and

$\Delta y_p(t)$, onto the reference great circles. This is easily done in practical applications and merely a technical question, which is beyond the scope of the present paper.

As a result we derive from the HIPPARCOS astrometric and photometric measurements the parameters D_x and D_y. This gives immediately the position angle Θ of the binary from

$$\tan\Theta = \frac{D_x}{D_y} \quad , \tag{25}$$

if x and y are equatorial coordinates and if Θ is measured from north through east. There is even no ambiguity by 180°, since we know that the vector $(D_x,\ D_y)$ points in the direction from the variable star 1 towards the constant star 2.

In contrast to the position angle Θ the distance d between the components can be determined only up to the factor F_2, the flux of the constant star 2, measured e.g. in terms of $\overline{F}$. A given amount of D,

$$D = (D_x^2 + D_y^2)^{1/2} = \frac{F_2}{\overline{F}}\, d \quad , \tag{26}$$

can be due either to a large value of d and a small value of F_2, or vice versa. We can derive, however, a lower limit d_{min} for d from the upper limit of F_2, namely $F_2 \le F_{min}$,

$$d \ge d_{min} = \frac{D}{(F_{min}/\overline{F})} = 10^{0.4(m_{tot,min} - \overline{m})} D \quad . \tag{27}$$

We cannot provide an upper limit for d in a similar way. For HIPPARCOS there is, however, a kind of practical upper limit of d. Firstly, binaries with $d > 100$ mas would be detected directly as visual binaries if their $\Delta m = m_2 - m_1$ is not too large. Secondly, HIPPARCOS is not measuring the photo-center of close binaries but the abscissae of a point which corresponds to the so-called 'combined' or 'weighted' phase (see Mignard et al. 1989 for details). The differences between the measured point and the photo-center is always smaller than 0.1 mas for $d < 40$ mas, and smaller than 1 mas for $d < 100$ mas. For $d \sim 200$ mas, the difference can reach about 10 mas. In this case, our method would not be any more very accurate.

In addition to a lower limit of d (Eq.(27)), we may also derive a lower limit of F_2 from the observed value of D. Since we normally know an upper limit d_{max} of d, beyond which the double star would be resolved as a visual binary, we have $F_2/\overline{F} \ge D/d_{max}$. In most cases, d_{max} itself depends, however, on the magnitude difference between the two stars, and hence on F_2 for a given F. Therefore, the lower limit of F_2 has to be determined iteratively. In summary, F_2 is then bounded to the interval $F_{min} \ge F_2 \ge \overline{F} D/d_{max}$.

The least-squares solution provides, beside the quantities D_x and D_y, also the positions $\overline{x}_p$ and $\overline{y}_p$. This is the position of the photo-center for the chosen reference flux $\overline{F}$ and therefore depends on the choice of $\overline{F}$. However, if we choose another value for the reference flux, say $\overline{F}'$, we can easily determine the corresponding position, $\overline{x}_p'$ (and similarily $\overline{y}_p{}'$), from Eq.(19):

$$\overline{x}_p' = \overline{x}_p + D_x \left(\frac{\overline{F}}{\overline{F}'} - 1 \right) \quad . \tag{28}$$

The quantities D_x and D_y are changed into

$$D_x' = D_x \frac{\overline{F}}{\overline{F}'} \quad , \tag{29}$$

which is simply due to the revised 'unit' of F_2 in Eq.(21).

As mentioned above, $\overline{F}$ should be a 'typical' value of F of the binary under consideration, e.g. the time average of $F(t)$. Otherwise, there would be a strong correlation between D_x and $\overline{x}_p$, and the formal mean error of $\overline{x}_p$ would be larger than necessary.

Nevertheless, it is interesting to determine $\overline{x}_p'$ for $\overline{F}' = \infty$. In that case, we obtain from Eqs.(18) and (28):

$$x_1 = \overline{x}_p'\,(\overline{F}' = \infty) = \overline{x}_p - D_x \quad . \tag{30}$$

This shows that we can actually determine the position x_1 (and y_1) of the variable star itself, without knowing d_x and x_2. But this position x_1 is strongly correlated with D_x, since it is usually an enormeous extrapolation of the observed positions of the photo-center during the limited variation in $F(t)$ to the formal limit $F \to \infty$. In practice, the position $\overline{x}_p$ for $\overline{F} \sim < F(t) >$ is to be preferred with respect to x_1. We should note that there is no way to determine x_2 in a similar way, because that would require the knowledge of F_2.

Equation (30) provides also an illustrative interpretation of the VIM 'elements' D_x and D_y: For example, D_x is the distance in x of the photo-center $\overline{x}_p$, for the chosen reference flux $\overline{F}$, from the variable star (located at x_1).

The determination of x_1 instead of $\overline{x}_p$ (and similarly of μ_1 according to Eq.(45) in Section 3) would be relevant, if the VIM is also a radio star and if the variable component is the source of the radiation at radio frequencies. Then, in a comparison of positions and proper motions at visual and radio wave-length, we should use x_1 and $\dot{x}_1$ rather than $\overline{x}_p$ and $\dot{\overline{x}}_p$.

While in principle any type of variable star in a binary system can produce a VIM, in practice we have certain requirements in addition to those discussed already in Fig.3: (1) The total flux $F(t)$ should vary 'sufficiently' in time between its maximal and minimal value, and (2) the time-scale of the variability should be longer than a few days and shorter than a few years. The first requirement ensures that our function $f(t)$ in Eq.(20) takes a large variety of values. Otherwise the least-squares solution would not be able to separate $\overline{x}_p$ and D_x. For example, eclipsing binaries are not well-suited for our method, because they provide a constant flux over most of the time (and because their separation, and hence the VIM effect, is essentially zero during the eclipse, if they are not a subcomponent in a triple system like Algol). Favourable for our method are Cepheids and Miras with their 'smooth' variability in time. They fulfill, in contrast to RR Lyraes, also the second requirement: The periods of RR Lyrae stars, of typically half a day, are too short in comparison to the typical periods of a few hours over which the astrometric data of HIPPARCOS are averaged in time during the standard data reduction procedure.

What happens if the star 2 is also a variable ? Then the binary would still behave as a VIM, showing motions of the photo-center. However, our quantitative treatment would fail. In order to handle such a case of two variables in a binary system, we would need the time variations of $F_1(t)$ and $F_2(t)$ separately, while HIPPARCOS provides only the sum of both, namely $F(t)$. If the stars behave quite 'normal', e.g. two Cepheids with different periods, it may be possible to decompose $F(t)$ into $F_1(t)$ and $F_2(t)$. Such a procedure would be interesting even outside of astrometric applications, and would also prove the binary nature of the system directly.

As we have shown it is not possible to determine the separation d of the components of a VIM directly. However, we may get a good estimate for d from the measured quantity $D = (D_x^2 + D_y^2)^{1/2}$, if we are able to derive the absolute magnitude of star 1 by astrophysical arguments. For example, if star 1 is a Cepheid, we can determine its absolute magnitude from the period-luminosity relation. Together with the astrometric or photometric parallax, this gives the (time-averaged) flux $< F_1(t) >$ for star 1, and hence from the observed total flux $< F(t) >$ the desired flux F_2 of the second component. This provides then the estimate $d = (\overline{F}/F_2)D$.

We should also note that the 'elements' D_x and D_y of a VIM depend on the photometric system used, since F is wave-length dependent. This is a similar situation as for the semi-axis major of the photo-center of a classical astrometric binary, which in general also depends on the photometric pass-band. For the main experiment of HIPPARCOS, there is no choice of the pass-band: D_x and D_y are in the HIPPARCOS photometric system Hp.

Up to now, we have only discussed whether or not an *individual* variable star can be identified as a VIM and hence as a close binary. However, even if the individual identification of VIMs may be difficult for many stars, the *collective* effect of VIMs on an ensemble of variable stars may nevertheless be *statistically* very significant. For example, even if the individual errors σ_D of D are rather large, say $(D/\sigma_D) \sim 2$, the statistical significance of an ensemble average (say $(< D^2 >^{1/2} / < \sigma_D^2 >^{1/2}) \sim 2$ for 100 objects) would give important statistical information about the frequency of binaries in this ensemble of stars.

3. Binaries with orbital motions

We shall now generalize the results of Section 2 to binaries with orbital motions. In this case, $d_x = d_x(t)$ and $d_y = d_y(t)$ change with time because of the motions of the two components relative to each other. Because of the orbital motions of both stars with repect to the center-of-mass, we have to discuss $x_1(t)$ and $x_2(t)$ in more detail. We denote the masses of the two stars by $\mathcal{M}_1$ and $\mathcal{M}_2$ and define

$$B = \frac{\mathcal{M}_2}{\mathcal{M}_1 + \mathcal{M}_2} \ . \tag{31}$$

Similarily, we define

$$\beta(t) = \frac{F_2}{F_1(t) + F_2} = \frac{F_2}{F(t)} \ . \tag{32}$$

The motions of the two stars in x (and similar in y) are given by:

$$x_1(t) = x_{cms,0} + \dot{x}_{cms}(t - t_0) - B d_x(t) \ , \tag{33}$$

$$x_2(t) = x_{cms,0} + \dot{x}_{cms}(t - t_0) + (1 - B) d_x(t) \ , \tag{34}$$

where $x_{cms,0}$ is the position of the center-of-mass of the binary at $t = t_0$ and $\dot{x}_{cms}$ is the constant velocity (i.e. the proper motion) of the center-of-mass. The common parallactic motion of both stars have still to be added to Eqs.(33) and (34) in practical applications. The motion of the photo-center is given by

$$\begin{aligned} x_p(t) &= (1-\beta) x_1 + \beta x_2 \\ &= x_{cms,0} + \dot{x}_{cms}(t - t_0) - (B - \beta(t))\, d_x(t) \ . \end{aligned} \tag{35}$$

The photo-center of the binary is still moving on the connecting line between the two stars, but this line is now rotating with the orbital motion of the binary.

3.1. Linear motion

Firstly we discuss the case where the changes in d_x and d_y can be approximated by linear functions:

$$d_x(t) = d_{x,0} + \dot{d}_{x,0}(t - t_0) \ , \tag{36}$$

$$d_y(t) = d_{y,0} + \dot{d}_{y,0}(t - t_0) \ . \tag{37}$$

Inserting Eq.(36) into Eq.(35) we obtain

$$\begin{aligned} x_p(t) &= \overline{x}_{p,0} + \dot{\overline{x}}_{p,0}(t - t_0) \\ &\quad + (D_{x,0} + \dot{D}_{x,0}(t - t_0)) f(t) \ , \end{aligned} \tag{38}$$

where

$$\overline{x}_{p,0} = x_{cms,0} - (B - \overline{\beta})\, d_{x,0} \ , \tag{39}$$

$$\dot{\overline{x}}_{p,0} = \dot{x}_{cms} - (B - \overline{\beta})\, \dot{d}_{x,0} \ , \tag{40}$$

$$D_{x,0} = \overline{\beta} d_{x,0} = \frac{F_2}{\overline{F}}\, d_{x,0} \ , \tag{41}$$

$$\dot{D}_{x,0} = \overline{\beta} \dot{d}_{x,0} = \frac{F_2}{\overline{\overline{F}}}\, \dot{d}_{x,0} \ , \tag{42}$$

$$\overline{\beta} = \frac{F_2}{\overline{\overline{F}}} \ . \tag{43}$$

$\overline{F}$ is the 'reference flux' and $f(t)$ is given by Eq.(20). Both $\overline{F}$ and $f(t)$ have already been used in Section 2. A least-squares solution of Eq.(38) and of the similar equation in y provides (beside the parallax) the position and proper motion of the photo-center($\overline{x}_{p,0}, \overline{y}_{p,0}, \dot{\overline{x}}_{p,0}, \dot{\overline{y}}_{p,0}$) for the reference flux $\overline{F}$, as well as the four quantities $D_{x,0}$, $D_{y,0}$, $\dot{D}_{x,0}$, $\dot{D}_{y,0}$. As in Section 2, we may derive the position and here also the proper motion of the variable star 1 by using $\overline{F}' = \infty$:

$$x_{1,0} = \overline{x}_{p,0} - D_{x,0} \ , \tag{44}$$

$$\dot{x}_{1,0} = \dot{\overline{x}}_{p,0} - \dot{D}_{x,0} \ . \tag{45}$$

We cannot, of course, determine the position and proper motion of the center-of-mass.

3.2. Accelerated motion

In this approximation, we add to Eq.(36) a quadratic term in t:

$$d_x(t) = d_{x,0} + \dot{d}_{x,0}(t - t_0) + \frac{1}{2}\,\ddot{d}_{x,0}(t - t_0)^2 \ . \tag{46}$$

We then obtain

$$\begin{aligned} x_p(t) \ &= \ \overline{x}_{p,0} + \dot{\overline{x}}_{p,0}(t - t_0) + \frac{1}{2}\,\ddot{\overline{x}}_{p,0}(t - t_0)^2 \\ &\quad +(D_{x,0} + \dot{D}_{x,0}(t - t_0) \\ &\quad +\frac{1}{2}\,\ddot{D}_{x,0}(t - t_0)^2)f(t) \ , \end{aligned} \tag{47}$$

where the new quantities are

$$\ddot{\overline{x}}_{p,0} \ = \ -(B - \overline{\beta})\,\ddot{d}_{x,0} \ , \tag{48}$$

$$\ddot{D}_{x,0} \ = \ \overline{\beta}\,\ddot{d}_{x,0} = \frac{F_2}{\overline{F}}\,\ddot{d}_{x,0} \ . \tag{49}$$

Here, we encounter the problem of a relation between the unknowns in the least-squares solution: The vector of acceleration ($\ddot{d}_x$, $\ddot{d}_y$) must be always anti-parallel to the positional vector (d_x, d_y), and ($\ddot{\overline{x}}_p$, $\ddot{\overline{y}}_p$) must be parallel or anti-parallel to ($\ddot{d}_x$, $\ddot{d}_y$). One may either build-in these relations into the least-squares solution, or one may use the relations as a physical check for the quality of the solution. Higher-order terms in Eq.(46) would increase this problem of interrelations between the unknowns, because the orbit $(d_x(t), d_y(t))$ is completely defined by seven independent orbital elements.

In contrast to the linear motion, the case of accelerated motion allows the determination of the position and proper motion of the center-of-mass of the binary. We denote by F_{cms} that value of the total flux F for which the photo-center is just identical with the center-of-mass. Such a value does always exist if $F_2 \neq 0$. F_{cms} may, however, be outside of the interval between F_{max} and F_{min} in which F is actually varying. In order to determine the center-of-mass, we change to a new reference flux $\overline{F}'' = F_{cms}$. From Eq.(48), we have

$$\begin{aligned} \ddot{\overline{x}}''_{p,0} \ &= \ \ddot{\overline{x}}_{p,0} + (\overline{\beta}'' - \overline{\beta})\ddot{d}_{x,0} \\ &= \ \ddot{\overline{x}}_{p,0} + \left(\frac{\overline{F}}{\overline{F}''} - 1\right)\ddot{D}_{x,0} \end{aligned} \tag{50}$$

with

$$\overline{\beta}'' = \frac{F_2}{\overline{F}''} = \frac{F_2}{F_{cms}} = \beta_{cms} = B \ . \tag{51}$$

The condition for $\overline{F}'' = F_{cms}$ is

$$\ddot{\overline{x}}''_{p,0} = \ddot{x}_{cms} = 0 \ , \tag{52}$$

because the center-of-mass of a two-body system is not accelerated. Using Eqs.(50) and (52), we obtain F_{cms} from

$$\frac{\overline{F}}{F_{cms}} - 1 = -\frac{\ddot{\overline{x}}_{p,0}}{\ddot{D}_{x,0}} \ . \tag{53}$$

The position and velocity of the center-of-mass follows from Eqs.(47) and (53) in analogy to Eq.(28):

$$x_{cms,0} \ = \ \overline{x}''_{p,0} = \overline{x}_{p,0} - D_x\frac{\ddot{\overline{x}}_{p,0}}{\ddot{D}_{x,0}} \ , \tag{54}$$

$$\dot{x}_{cms} \ = \ \dot{\overline{x}}''_{p,0} = \dot{\overline{x}}_{p,0} - \dot{D}_x\frac{\ddot{\overline{x}}_{p,0}}{\ddot{D}_{x,0}} \ . \tag{55}$$

It is useful to replace $D''_{x,0}$, which is determined in analogy to Eq.(29), by a new quantity $d_{x,1}$,

$$\begin{aligned} d_{x,1}(t) \ &= \ x_1(t) - x_{cms}(t) = -Bd_x(t) \\ &= \ -\beta_{cms}\,d_x(t) = -D''_x(t) \ , \end{aligned} \tag{56}$$

which describes the distance in x of the variable star 1 from the center-of-mass. We find

$$d_{x,1,0} \ = \ -D''_{x,0} = -D_{x,0}\frac{\overline{F}}{F_{cms}} = D_{x,0}\left(1 - \frac{\ddot{\overline{x}}_{p,0}}{\ddot{D}_{x,0}}\right) \ , \tag{57}$$

$$\dot{d}_{x,1,0} \ = \ -\dot{D}''_{x,0} = -\dot{D}_{x,0}\frac{\overline{F}}{F_{cms}} = \dot{D}_{x,0}\left(1 - \frac{\ddot{\overline{x}}_{p,0}}{\ddot{D}_{x,0}}\right) \ , \tag{58}$$

$$\ddot{d}_{x,1,0} \ = \ -\ddot{D}''_{x,0} = -\ddot{D}_{x,0}\frac{\overline{F}}{F_{cms}} = \ddot{D}_{x,0} - \ddot{\overline{x}}_{p,0} \ . \tag{59}$$

From Eqs.(54–55) and (57–59), we can also derive the position $x_1(t)$ of the variable star by

$$x_1(t) = x_{cms}(t) + d_{x,1}(t) \ . \tag{60}$$

Especially we have

$$\ddot{x}_{1,0} = \ddot{d}_{x,1,0} = \ddot{D}_{x,0} - \ddot{\overline{x}}_{p,0} \ , \tag{61}$$

in analogy to Eqs.(44) and (45).

The method of determining F_{cms} by Eq.(53) has a disadvantage: We have a similar equation in y, and due to measuring errors, the equations in x and y will give two different values for F_{cms}. We may, of course, take a suitable mean of both as the finally adopted value of F_{cms}. However, there does exist a method which avoids this procedure. We rewrite Eq.(35), using $\beta_{cms} = B$ and $d_{x,1}(t) = - \beta_{cms} d_x(t)$:

$$\begin{aligned} x_p(t) &= x_{cms,0} + \dot{x}_{cms}(t - t_0) - (\beta_{cms} - \beta(t)) d_x(t) \\ &= x_{cms,0} + \dot{x}_{cms}(t - t_0) + d_{x,1}(t) \left(1 - \frac{\beta(t)}{\beta_{cms}}\right) \\ &= x_{cms,0} + \dot{x}_{cms}(t - t_0) \\ &\quad + (d_{x,1,0} + \dot{d}_{x,1,0}(t - t_0) \\ &\quad + \frac{1}{2}\ddot{d}_{x,1,0}(t - t_0)^2) \left(1 - \frac{F_{cms}}{F(t)}\right) \ . \end{aligned} \quad (62)$$

A similar equation holds in y. The quantity F_{cms} is now to be considered as an additional unknown, common to the equations in x and y, which has to be determined by a simultaneous least-squares solution of Eq.(62) and of the corresponding equation in y for F_{cms}, $x_{cms,0}$, $y_{cms,0}$, $\dot{x}_{cms}$, $\dot{y}_{cms}$, $d_{x,1,0}$, $d_{y,1,0}$, $\dot{d}_{x,1,0}$, $\dot{d}_{y,1,0}$, $\ddot{d}_{x,1,0}$, $\ddot{d}_{y,1,0}$, and the parallax. This least-squares solution is unfortunately not any more linear because of F_{cms}. As before, the vector $(\ddot{d}_{x,1,0}, \ddot{d}_{y,1,0})$ should be anti-parallel to $(d_{x,1,0}, d_{y,1,0})$.

3.3. Keplerian motion

For astrometric binaries with a period P between about a month and a few years, HIPPARCOS is often able to measure the full Keplerian orbit of the photo-center with respect to the center-of-mass. Here we discuss the additional complication if one of the components is a variable star.

We describe the relative orbit of the two components by

$$d_x(t) = a_{12}\, s_x(t) \ , \quad (63)$$
$$d_y(t) = a_{12}\, s_y(t) \ , \quad (64)$$

where a_{12} is the (unknown) semi-axis major of the relative orbit of star 2 with respect to star 1, and $s_x(t)$ and $s_y(t)$ are well-known (e.g. Heintz 1978) functions of time t, which describe the orbital motion of a double star, if the remaining six orbital elements (e.g. period P, eccentricity e, periastron time T, periastron length ω, nodal line Ω, inclination i) are given. Inserting Eq.(63) into Eq.(35), we obtain

$$\begin{aligned} x_p(t) &= x_{cms,0} + \dot{x}_{cms}(t - t_0) \\ &\quad -(B - \beta(t))\, a_{12}\, s_x(t) \\ &= x_{cms,0} + \dot{x}_{cms}(t - t_0) \\ &\quad -\overline{\alpha}\, s_x(t) \\ &\quad +A\, s_x(t)\, f(t) \ , \end{aligned} \quad (65)$$

where

$$\overline{\alpha} = (B - \overline{\beta})\, a_{12} \ , \quad (66)$$
$$A = \overline{\beta}\, a_{12} = \frac{F_2}{\overline{F}}\, a_{12} \ . \quad (67)$$

$\overline{\alpha}$ is the semi-axis major of the orbit of the photo-center for the reference flux $\overline{F}$, and A is a quantity which corresponds to D in the case of a fixed geometry. Together with the position, proper motion and parallax of the center-of-mass, and some or all of the six elements required for $s_x(t)$ and $s_y(t)$, a least-squares solution of Eq.(65) and of a similar equation for $y_p(t)$ will provide $\overline{\alpha}$ and A for an astrometric binary with a variable component. From Eqs.(66) and (67), we derive

$$a_1 = B\, a_{12} = \overline{\alpha} + A \ . \quad (68)$$

a_1 is the semi-axis major of the orbit of the variable star with respect to the center-of-mass. If the system is already known as a spectroscopic binary, the same quantity a_1 (in mas) is known from the spectroscopic element a_1 sin i (in km) and the astrometric inclination and parallax. This can be either used as a check for $\overline{\alpha}$ and A by Eq.(68). Or the knowledge of a_1 can be used to determine A in the case of a low-amplitude variability or of an eclipsing binary by

$$A = a_1 - \overline{\alpha} \ . \quad (69)$$

Such an A, determined from Eq.(69), could be used to reduce (iteratively) all the observations to the reference flux $\overline{F}$, and to apply then the standard method for non-variable astrometric binaries to the system.

3.4. Triple and multiple stars

In the former sections, we have always assumed that the system under consideration consists of two stars only. In the case of a higher-order (triple or multiple) system, the methods have to be modified only slightly.

In the case of a fixed geometry between all the components of the system, only the interpretation of the elements D_x and D_y changes. F_2 is to be replaced by the integrated flux of all the constant components, and d_x is the distance in x of the photo-center of these constant components from the variable star 1. Section 3.1 (Linear motion) has to be generalized in the same way. In Sections 3.2 and 3.3 (Accelerated or Keplerian motions), the center-of-mass may have to be replaced by an accelerated reference point, if the gravitational attraction by the other bodies does require such an extension of the model.

4. Colour-induced displacements of the photo-centers of binaries (CIDs)

For double stars which do not contain a variable component, their binary nature may still be detected by means of another ‘anomalous’ behaviour of the photo-center, if the observations are carried out at different wavelength and if the colours of the components differ. Let us assume that the observing instrument (e.g. GAIA) observes the position (or RGC abscissa) of the photo-center of a close binary in two different photometric pass-bands (with central wavelenght λ_I and λ_{II}). If the colours, i.e. essentially the ratios $F_{\lambda I}/F_{\lambda II}$, of the two components differ, then the position of the photo-center depends on the pass-band (i.e. on λ). For example, if the two components have the same magnitude in V ($V_1 = V_2$), but a colour difference $\Delta(B-V) = 1^m$ (i.e. $B_2 = B_1 + 1^m$), then the photo-center in V is just at the mid–point between the two components, while the photo-center in B is shifted towards the bluer star 1 and has a distance from the star 1 of 28 % of the total separation d. Hence the difference between the photo-center in B and V is 22 % of d in our example. If both components are main-sequence stars, the difference between the photo-centers in B and V is typically between 1 % and 4 % of d for $\Delta(B-V)$ in the range between 0.1 and 1.0 (with the maximum near $\Delta(B-V) \sim 0.4$). Such differences should be measurable in many cases.

If the relative geometry of the binary is fixed (as assumed in Section 2 for VIMs), the displacement of the two photo-centers in λ_I and λ_{II} is also fixed and is described by two constants, $C_x = x_p(\lambda_{II}) - x_p(\lambda_I)$ and $C_y = y_p(\lambda_{II}) - y_p(\lambda_I)$. If the observations are carried out in more than two pass-bands, all the different positions of the photo-centers of a binary should be lined-up on a part of a straight line, connecting the two components. Especially this property would be an unmistakable evidence for the binary nature of an object. Multiple stars would show more complicated patterns. As for VIMs, the displacement of the photo-centers would determine the position angle Θ, but not the separation d. The generalisation to binaries with orbital motions, as carried out in Section 3 for VIMs, is straight-forward. For example, in the case of a linear motion of the two components relative to each other, the velocity of the photo-center with respect to the center-of-mass would depend on λ, because of the λ dependence of the distance of the photo-center from the center-of-mass. The result would be that the total proper motion of the photo-center (including the center-of-mass motion of the system) would also depend on the pass-band, i.e. on λ.

A quantitative modelling of the displacements of the photo-center as a function of λ in the case of more than two pass-bands does not seem to be strictly possible, due to the various spectral distributions of stellar fluxes. In practice, a well-chosen approximation in which Δx_p is a suitable function of λ may be sufficient.

Unfortunately for the CID method, the main experiment of HIPPARCOS is working in one pass-band (Hp) only. TYCHO (the additional experiment on board of the HIPPARCOS satellite) which actually observes in two pass-bands, B_T and V_T, would in principle be able to make use of the CID method. For the GAIA project, many pass-bands are foreseen, and hence the CID method should be applied in the data reduction for GAIA.

5. Conclusions

We have shown that the photo-center of a binary in which one component is variable moves in a very characteristic manner as a VIM. For a binary with a fixed relative geometry, the photo-center moves back and forth on part of a straight line connecting the two components. The motion is quantitatively determined by the observable total flux $F(t)$ of the system, and by two constants, D_x and D_y, which represent the basic 'elements' of a VIM and are defined by Eqs.(21) and (22). The motion of the photo-center is proportional to the separation d of the system, to the flux F_2 of the constant star, and is a linear function of the inverse of the varying total flux $F(t)$ of the system (Eq.(20)). In the case of an orbital motion of the binary, the situation can also be described quantitatively (Section 3).

The full correlation of the motion of a VIM with its variable total flux $F(t)$ provides in most cases an unmistakable evidence for the binary nature of this system, since other astrometric motions of stars are in general quite different from that of a VIM and are especially not in the same rhythm as the light variations of the stars.

Since most of the common, non-variable stars are members of double stars, this should be also true for variable stars. If the amplitude of the light variation (Δm_{tot}) and the separation d of the binary are large enough and if the constant component is not too faint with respect to the variable star, then the system will show a measurable motion of the photo-center as a VIM. Hence all the variable stars observed by HIPPARCOS should be tested as to whether or not they are VIMs. We have shown in Section 2 and 3 how that can be done quantitatively. We expect to find a significant number of VIMs among the variable stars observed by HIPPARCOS. Even if the binary nature of some VIMs is already known from other indicators, they should be treated as VIMs, because otherwise the high measuring accuracy of HIPPARCOS is not fully used for the standard astrometric parameters (position, proper motion, parallax) because of modelling errors.

The testing of variable stars for being VIMs would be even more important for extremely accurate astrometric missions like the GAIA project, because otherwise the modelling errors of standard methods (instead of the VIM method) would be much larger than the very small measuring errors of GAIA and would for example weaken the quality of the parallaxes of many of the variable stars (Cepheids!) significantly. The additional effort for applying the VIM method is very small in relation to the possible gain in accuracy and in internal consistency.

Another special behaviour of the photo-center of a double star could also decover the binary nature of an object: If the observations are carried out in at least two photometric pass-bands and if the two components differ in colour, then the position and the motion of the photo-center depend in a characteristic way on the central wavelength λ of the pass-band. We propose to call such an object a 'CID', because its photo-center shows a 'colour-induced displacement'. The method to detect and to handle CIDs is outlined in Section 4. Similar to the VIM method, the CID method does not only allow to detect binaries, but also to improve the astrometric treatment of CIDs (with respect to a standard treatment as single stars). Hence e.g. all GAIA objects should be tested whether or not they show the properties of CIDs, and should be treated as such if they are actually detected as CIDs.

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