How good is Superbox ?

Michael Fellhauer, e-mail: mike@ari.uni-heidelberg.de

Astronomisches Rechen-Institut Heidelberg

1 Introduction

The majority of the known self-consistent methods to simulate self-gravitating point-mass systems have historically not been useful for studying systems with small filling factors such as interacting and merging galaxies or groups of galaxies (Barnes+Hernquist 92 [B+H92]). Exceptions are the hierarchical tree method (Barnes+Hut 86 [B+H86]) and multi-grid or nested grid particle mesh-codes like Hydra (Pearce+Couchman 97 [P+C97]) and Superbox (Bien [B+W91] and articles in this volume). Superbox is a particle-mesh program with high-resolution sub-grids to simulate galaxies and groups of galaxies. In this talk the conservation of energy and angular momentum of Superbox is investigated and the memory and CPU-time consumption are profiled.

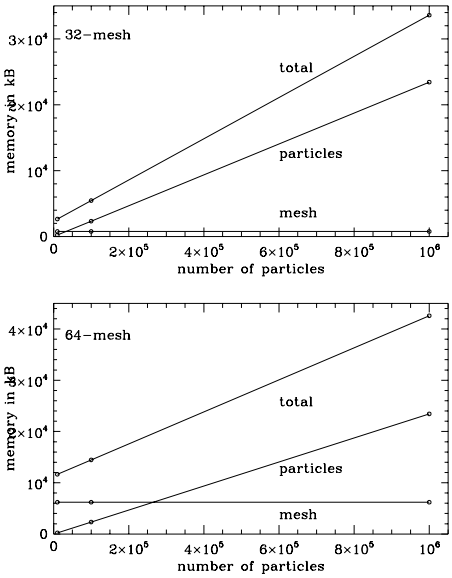

Figure 1: Memory requirement of Superbox; total storage and the fractions for the mesh and the particles are plotted for 32- and a 64-grids respectively

2 Memory + CPU-Time

2.1 Memory

The memory requirements of Superbox scale both linear in particle number and in the number of grid-cells.
If one changes the resolution from 32 to 64 cells per grid-dimension (32-grids, 64-grids) the amount of memory increases with a factor of eight. The 32-grids need 780 kB of storage, the 64-grids 6 MB. And working with 128-grids would need 48 MB. For the particle array 24 byte per particle is needed.
The total amount of memory for different particle numbers and grid-resolutions is shown in figure 1. If one simulates more than one galaxy one should note that the memory usage of the grids is the same as for one galaxy. No extra storage is needed because all galaxies are treated in the same grid-arrays one after another. Increasing the galaxy number while keeping the total particle number constant reduces the total amount of storage slightly. This is due to the fact that some arrays scale with the number of particles per galaxy which then are shorter.

2.2 CPU-time

The time-step cycle of Superbox is divided in three main sequences. First, there is the the *getrho* routine where the mass density of the grids is derived.

Second the *fft* routines calculating the potentials of the grids via Fast Fourier Transform and third the *pusher* routines containing force calculation, integration of particles and collecting the output data. These three routines need about 99 % of the total CPU time. Figure 2 shows the amounts of CPU-time per step needed for the different routines. All data is derived on a Pentium 200MMX simulating a single galaxy. While getrho and pusher routines scale linearly with the particle number the fft routine scales with the number of grid-cells. As like the memory consumption a change from 32 to 64 cells per dimension increases CPU-time for the fft by a factor of 8.
To get the total amount of CPU-time per step one simply has to add the three corresponding figures in 2. Also, it can be seen, most of the time is spent for the fft-routines.
As a example one million particles simulated with 64-grids take less than 20 seconds on the Pentium.
The CPU-times for more than one galaxy can be derived in adding up the times for each galaxies alone.

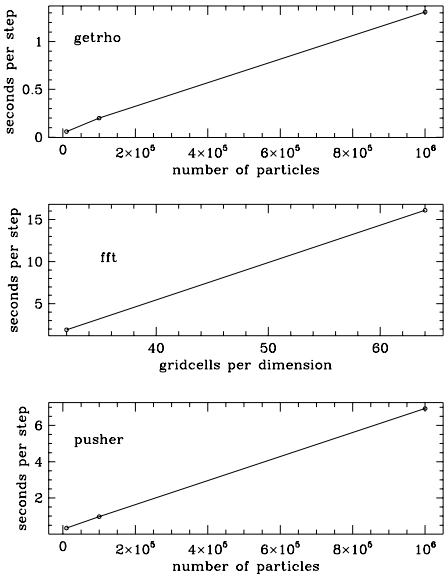

Figure 2: CPU time of the different routines of Superbox; top: getrho scales with the number of particles, middle: fft scales with number of grid-cells, bottom: pusher scales with particle number

3 Energy and Angular Momentum

The calculation of energies and angular momentum is done only approximately during runtime. For calculating the potential energy only the potential of the cells in which the particle is located is taken into account. This is just a crude approximation to save CPU-time. Therefore, if a galaxy moves from one grid-cell into another in the outer grid (so called "local universe") you see oscillations in potential energy which are an artefact of the energy calculation. In contrary to this approximation the force calculation is done with a second order Taylor-polynomial taking into account the deviation of the particle from the center of the cell. Also, because of the Leap-frog scheme, the velocities stored in Superbox are one half of a time-step behind the positions. In the old Superbox version the kinetic energy was calculated using these velocities, implying an error of the order Δv. Now kinetic energies are calculated before and after updating the velocities and the average value is taken. This is still not exact but this procedure implies only an error of the order of $(\Delta v)^2$.
The old version of Superbox has had no routine to calculate angular momentum at all. Now the same technique as for kinetic energies is applied. Namely averaging the angular momenta before and after the velocity-integration.
All simulations presented here are performed with Plummer-spheres for 1,000 time-steps with a Δt of $T_{\rm cr}/100$. This means 10 internal crossing-times ($T_{\rm cr}$) of the Plummer-sphere.

3.1 Total Energy

First test runs with different time-steps were done. Table 1 shows no effect on the conservation of energy if the time-step is chosen properly. Only with an obviously

too large time-step of $T_{\rm cr}/10$ one looses accuracy.

Δt	$\Delta E_{\rm max}$
$T_{\rm cr}/10$	0.4 %
$T_{\rm cr}/100$	0.2 %
$T_{\rm cr}/1,000$	0.2 %

Table 1: Maximum relative change in $\Delta E_{\rm max}$ [%] due to different time-steps; 1 galaxy with 100,000 particles on 32-meshes over 10 crossing-times.

Next, the effects of different particle numbers and grid-resolutions were investigated. Runs with 10^4, 10^5 and 10^6 particles on 32 an 64 grid-resolutions were performed.

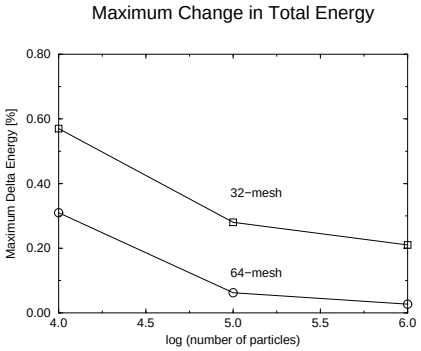

Figure 3: Maximum relative change in total energy [%] over 10 crossing-times, for different particle numbers and grid-resolutions

Increasing the number of particles to more than 100,000 particles per galaxy show only a little improvement of the conservation of energy. But changing the grid resolution from 32 to 64 cells per dimension gives an improvement of a factor of at least 4. Details are shown in figure 3.
Figure 4 shows the time evolution of the change in total energy for a particular run. This run was performed with one million particles on 64-grids. A linear decrease of the energy fits the data well. Even in simulations performed over 50 crossing-times the change in energy was still linear. The slope of the energy-loss in this run is 0.002 % per crossing-time.

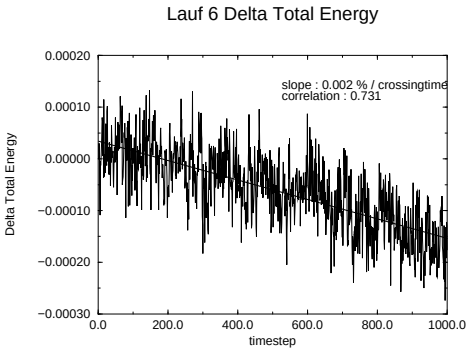

Figure 4: Linear regression of the relative change in total energy ($\Delta E/E$) over 10 $T_{\rm cr}$ (Lauf6)

3.2 Angular Momentum

The conservation of angular momentum does not depend on the time-step size at all. But it is highly dependent on the grid-resolution. Changing from 32 to 64 grids improves the change in angular momentum of at least a factor of 10. One can also see in figure 5 that the conservation of angular momentum does not depend on particle number as well. Figure 5 shows the maximum deviation in angular momentum in percent over 10 $T_{\rm cr}$ for the different particle numbers and grid resolutions.

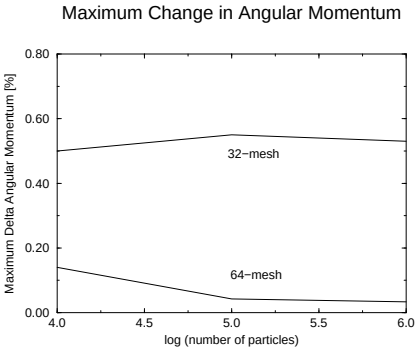

Figure 5: Maximum change in angular momentum [%] over 10 crossing-times, for different particle numbers and grid-resolutions

Looking also on the time evolution of these changes at a particular run (figure 6), performed with 1 million particles on 64-grids shows again that a linear decrease fits the data nicely. As in the

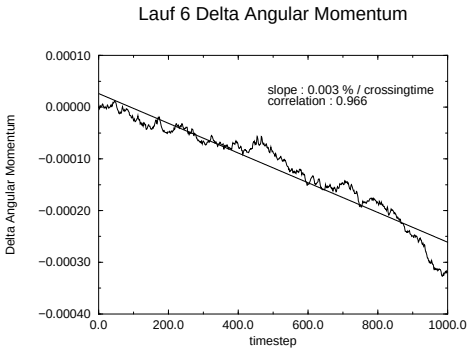

Figure 6: Linear regression of the relative change of angular momentum ($\Delta A/A$) over 10 $T_{\rm cr}$ (Lauf6)

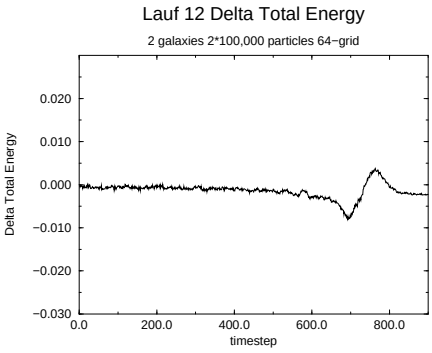

Figure 7: Lauf12: Relative change in total energy ($\Delta E/E$), 2*100,000 particles on 64-mesh

case of the total energy, simulations over 50 crossing-times showed no deviations from this linear decrease. The slope of the fitting line is 0.003 % per crossing-time.

4 More than One Galaxy

Simulations of more than one galaxy suffer from the high dependence of the conserved quantities on the resolution of the outermost grid (the so-called "Local Universe"). Although one gets the same results in morphology and velocity distribution the changes in energy and angular momentum can sum up to a few percent if the outer grid is chosen improperly. As explained above artificial oscillations in total energy can be seen if galaxies move from one grid-cell into another. These oscillations are artefacts of the approximated calculation of potential energy and are no real errors of Superbox.
But during a close encounter of two galaxies the high resolution grids of both galaxies do overlap. Therefore particles of the galaxies feel the high resolution forces of both galaxies. This is an additional source of error for energy and angular momentum.
As said before these effects can be minimised by keeping the "Local Universe" as large as necessary but as small as possible and choosing the resolution of the grids properly.

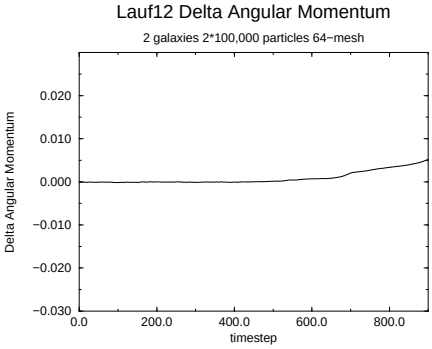

Figure 8: Lauf12 Delta Angular Momentum, 2×100,000 particles on 64 mesh

The two figures above (7 + 8) show the changes in the energy and angular momentum during a particular run. This run was performed with 100,000 particles per galaxy and 64 grids. It is a fast nearby encounter of two Plummer models. At the point of closest approach the galaxies are less than one Plummer scale-length separated. Therefore even the very high resolution grids of the galaxy cores do overlap at this time. One still can see the artifical oscillation and after the time of closest approach (around time-step 700) the total energy value has been shifted slightly (about 0.5 %). So having in mind that the transformation of orbital energy of the galaxies into internal energy can amount up to about 50 % of the initial internal energy of one galaxy such an error is quite tolerable.
As one can see the angular momentum increases slowly after the encounter. Af-

ter all this is also only an effect of some 0.5 %.

5 Conclusions

Superbox is a highly efficient tool to simulate one or many galaxies. The code is very fast, needs a low amount of memory and is able to treat an arbitrary number of galaxies fully self-consistently. The changes in total energy and angular momentum are excellent when treating an isolated galaxy and they are reasonably well in interacting systems of two and more galaxies. Due to the fact that one can simulate with high particle numbers and a high resolution at the galactic cores detailed morphological effects can be resolved with low noise.
Additional confidence into Superbox gives the fact, that results derived with Superbox show no differences to the results gained with other codes (see Ralf Klessen in this volume and Klessen+Kroupa [K+K97]).

References

[B+H92] J.E. Barnes, L. Hernquist: Dynamics of interacting galaxies; A&A Vol.30, p.705-742

[B+H86] J.E. Barnes, P. Hut: Error analysis of a tree-code; ApJ-Sup Vol.70, p.389-417

[P+C97] F.R. Pearce, H.M.P. Couchman: Hydra: A Parallel Adaptive Grid Code; NewA Vol.2, p.411-427

[B+W91] R. Bien, B. Fuchs, R. Wielen: High spatial resolution using the conventional particle-mesh technique; The CP90 Europhysics Conference on Computational Physics, Serie A, No.228, p.3-13

[K+K97] R. Klessen, P. Kroupa: paper in preperation

Superbox is available under:
ftp://auriga.ari.uni-heidelberg.de/
pub/mike/super.tar.gz