What is Superbox and what is it for ?

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1 Introduction

Superbox is an improved particle-mesh code with high resolution sub-grids, which can be focused either on the center of mass or the center of density of the galaxies. Superbox is able to treat an arbitrary number of galaxies. Our code is a very fast and low-storage algorithm, running even on small workstations and PCs (see Michael Fellhauer in this volume).

2 Method

Superbox is basically a particle-mesh code where the mass-densities are derived from cartesian grids. From these densities the potential is calculated via very fast discrete FFT-algorithm (FFT = Fast Fourier transform). This grid-based mean-field potential is differentiated numerically using a second order Taylor polynomial, where the deviations of a particle from the center of the host cell is taken into account. In this way one gets the accelerations of each particle, which is then integrated with the Leap-frog scheme (see: [B1])

2.1 Greens-Function

To derive the potential from the mass-densities one simply multiplies the Fourier-transformed density-grid with a Fourier-transformed Greens-function. The Greens-function for these particular problem, namely cartesian coordinates with cell-lengths of 1, looks like:

$$\begin{aligned} H(i,j,k) &= \frac{1}{\sqrt{i^2+j^2+k^2}} \quad i,j,k = 0...n \qquad (1) \\ H(0,0,0) &= \frac{1}{0.75} \end{aligned}$$

H is the array of the Greens-function, n is the number of grid-cells per dimension and has to be a power of 2

The value of $H(0,0,0)$ is taken from tests done by D. Pfenniger ([P1]) and P. Kroupa ([K1]) and counts for the forces between the particles in a common cell.

2.2 Mass-Density

The mean densities of the cells is calculated by a simple particle-in-cell algorithm. In other words the mass of a particle is added totally to that cell where the particle is in (see figure 1). A cloud-in-cell technique, where the mass of the particle is smeared out to the neighbouring cells seems to be not necessary, since in our code the number of particles is extremely high. Though there is no need to loose resolution beyond one cell-length.

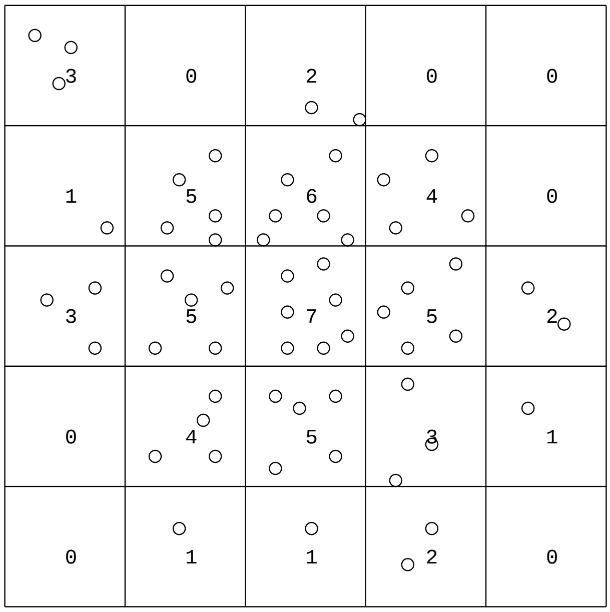

Figure 1: Example how the mass-density is derived in the grid-cells

2.3 FFT

Our FFT-routine is taken from Praktische Mathematik II ([S1]). As far as we know this algorithm is faster than other FFT-routines taken from standard Fortran liberies. The low-storage algorithm to get the potential in three dimensions is taken from F. Hohl ([H1]); see also [H2].

2.4 Acceleration + Integration

Once the grid-based potentials are derived they are differentiated numerically using a second order Taylor polynomial in order to get the acceleration for each particle:

$$
\begin{aligned}
a_x(i,j,k) \quad = \quad & \frac{\Phi(i+1,j,k)-\Phi(i-1,j,k)}{\Delta x} \\
& + \frac{\Phi(i+1,j,k)+\Phi(i-1,j,k)-2\cdot\Phi(i,j,k)}{(\Delta x)^2}\cdot dx \\
& + \frac{1}{2}\left(\frac{\Phi(i+1,j+1,k)-\Phi(i-1,j+1,k)}{(\Delta y)^2}\right. \\
& \left. + \frac{\Phi(i-1,j-1,k)-\Phi(i+1,j-1,k)}{(\Delta y)^2}\right)\cdot dy \\
& + \frac{1}{2}\left(\frac{\Phi(i+1,j,k+1)-\Phi(i-1,j,k+1)}{(\Delta z)^2}\right. \\
& \left. + \frac{\Phi(i-1,j,k-1)-\Phi(i+1,j,k-1)}{(\Delta z)^2}\right)\cdot dz
\end{aligned} \tag{2}
$$

dx, dy, dz denote deviation of the particle from the center of the cell in x, y or z; Δx, Δy, $\Delta z = 2$ because the cell-length is unity; the accelerations in y- and z-direction are calculated similarly.
Then the particles are integrated using the Leap-frog scheme.

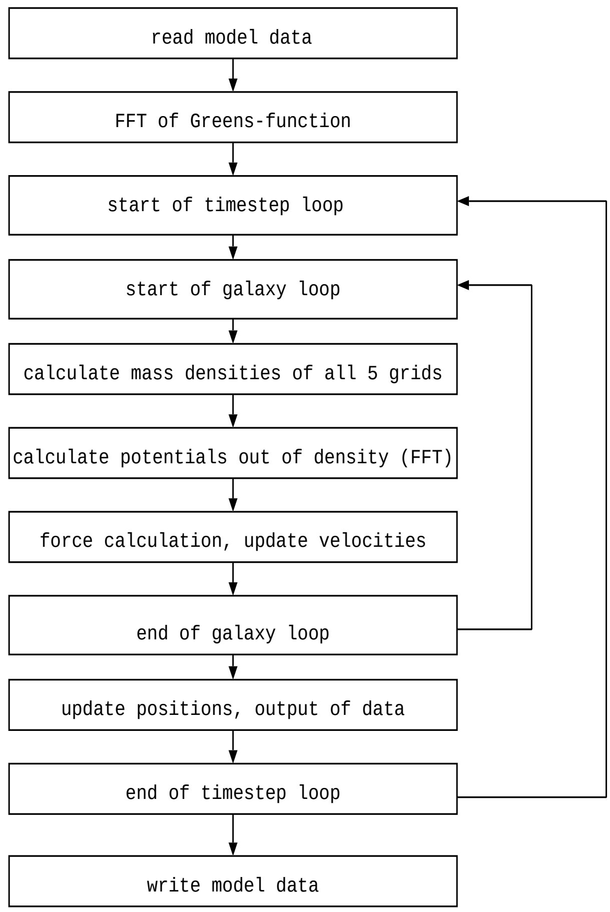

Figure 2: Working diagram of Superbox

3 The Grids

The main advantage of Superbox are the high resolution sub-grids. Galaxies are not only treated in one low-resolution grid covering the total simulation area (=”local universe”). For each galaxy Superbox uses 5 grids with 3 different resolutions:

- Grid 1 is the high resolution grid that can resolve the centers of the galaxies. It has the length $2 \times rcore$. In evaluating the densities, all particles of the galaxy within $r \leq rcore$ are stored in this grid.
- Grid 2 is a grid with mean resolution to resolve the galaxy as a whole. The length is $2 \times rout$, but only particles with $r \leq rcore$ are stored here, corresponding to grid 1.
- Grid 3 has the same size and resolution like grid 2, but in this grid the particles with $rcore < r \leq rout$ are stored.
  One has to keep in mind that the grids 1 to 3 are focused on the galaxy and are able to move through the ”local universe”.
- Grid 4 has the size of the ”local universe” and has the lowest resolution. It is fixed and not able to move. But only particles of the galaxy with $r \leq rout$ are stored in grid 4.

- Grid 5 has also a low resolution and is fixed too. The size is of the "local universe" $= 2 \times rsystem$. This grid treats the escaping "stars" of a galaxy (e.g. due to an interaction) and stores all particles with $r > rout$.

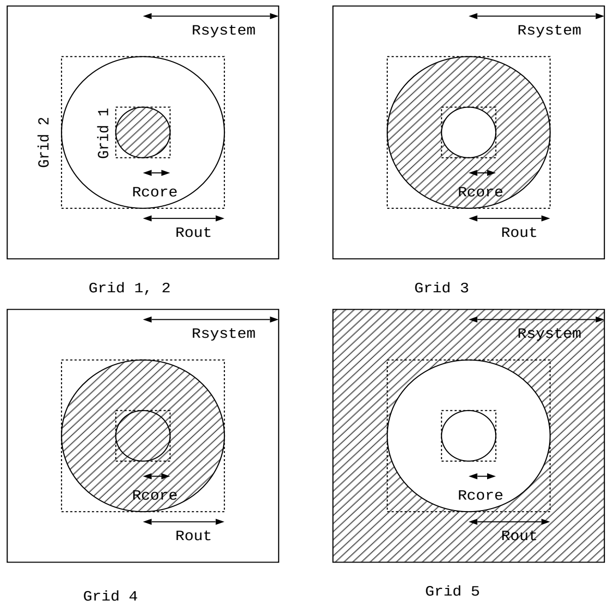

Figure 3: The five grids of Superbox; particles are counted in the shaded areas of the grids only

To keep the storage low all galaxies are treated in the same grid-arrays one after another!
Therefore the Leap-frog scheme has to be splitted. First the 5 potentials of galaxy 1 are calculated. Then the accelerations of all particles (of all galaxies) are derived as follows:

- Is a particle in the range $r \leq rcore$ the potentials of grids 1, 3 and 5 are used to calculate the acceleration.
- Between $rcore < r \leq rout$ the potentials of grids 2, 3 and 5 are needed.
- And last if $r > rout$ the acceleration is calculated from potentials of grids 4 and 5.
- A "star" with $r > rsystem$ is ruled out of the simulation.

This implies that as long as the galaxies are well separated they "feel" only the "crude" potentials of the outer grids. But as the galaxies approach each other their high-resolution grids do overlap leading to a high-resolution force-calculation during the interaction. No approximation to some multi-pole expansion is needed because the fft-algorithm gives the "exact" solution of the grid potentials. With these accelerations (due to the potential of galaxy 1) the velocities of all stars are integrated (via Leap-frog). Then the potentials of galaxy 2 are calculated and again the accelerations for all particles are derived and the velocities are updated. When this is done for all simulated galaxies the positions of all particles are integrated (see figure 2).
The splitting of the potentials is due to the additivity of the potentials.

4 Conclusions

This shows that Superbox is a fully self-consistent code which is able to treat large particle numbers (up to several millions without the need of a parallel supercomputer) in interacting galaxies with an arbitrary number of galaxies or galaxy components (disk, bulge, dark halo...) where is each treated with focused higher resolution grids. And the centers of the galaxies with even highest resolution.
Because of the grid-based technique (mean-field) two-body-relaxations are neglectable. Tests show that such effects become significant after several Hubble-times only. The limits of a particle-mesh code is only defined by the size of the grid-cells. You can not resolve features smaller than a cell-length.
Simulations done with Superbox are in good agreement with observations:

- R. Bien, R. Madejski: The high-velocity encounter of NGC 4782/4783: comparison of numerical experiments and observations Astron. Astrophys. 280,383-399; 1993
- N. Wassmer, R. Bien, B.Fuchs, R. Wielen: Close Encounters of Elliptical Galaxies: An Interpretation of the Morphological Asymmetry; Astron. Ges. Abstr. Ser. 9, 78; 1993
- R. Bien, N. Wassmer, R. Wielen: Results on the evolution of small groups of galaxies using the Superbox code; XXIInd General Assembly of the IAU Den Haag, The Netherlands, 15.-27. August 1994, Astronomy Poster Abstracts, H. van Woerden (ed.) Joint Discussion 5, p.202; Twin Press Sliedrecht; 1994
- N. Wassmer, R. Bien, R. Wielen: Massive black holes in interacting galaxies; R. Bender, R. L. Davies (eds.): New light on galaxy evolution, Proc. IAU Symp. 171 Heidelberg, Germany 26.-30. June 1995; Kluwer Dordrecht
- P. Kroupa: Dwarf spheroidal satellite galaxies without dark matter (NewA 2,139-164; 1997)

The article of R. Klessen in this volume (and [K2]) shows also that results derived with Superbox show no differences to other codes.

References

[B1] R. Bien, B. Fuchs, R. Wielen: High spatial resolution using the conventional particle-mesh technique; The CP90 Europhysics Conference on Computational Physics, Serie A, No. 228, p. 3-13

[P1] D. Pfenniger: private communication

[K1] P. Kroupa: private communication

[S1] H. Werner, R. Schabach: Praktische Mathematik II, Springer Verlag 1979

[H1] F. Hohl: Dynamical evolution of disk galaxies, NASA Technical Report R-343, 1970

[H2] R. W. Hockney, J. W. Eastwood: Computer Simulations Using Particles, McGraw-Hill 1981

[K2] R. Klessen, P. Kroupa: paper submitted